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LINEAR PROGRAMMING

AND

SHORT TERM FINANCIAL PLANNING

by

CHARLES W. YOUNG

A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Business Administration

Graduate School of Business Administration
George F. Baker Foundation
Harvard University

February 1968

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ABSTRACT

LINEAR PROGRAMMING AND SHORT TERM FINANCIAL PLANNING

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Graduate School of Business Administration
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The purpose of this research was to test and evaluate the practical usefulness of linear programming as a management tool for short term financial planning. Two actual firms, identified as COMPA and COMPB, were used as subjects for these tests.

This study had three objectives. The first was to determine if information routinely accumulated in a firm can be used to formulate a fully integrated linear programming model. Successful development of models of both COMPA and COMPB indicates that if a firm satisfies the linear programming economic criteria it may be possible to develop a model from data available in company records.

The second objective was to establish that the models were representative of the subject firms. Simulation tests established that the COMPA fully integrated multi-period model was adequately representative. However, shortcomings in the quality of data used to formulate the COMPB model prevented it from adequately representing the firm. The final objective was to validate or qualify the usefulness of a representative fully integrated model for short term financial planning. This phase of the study used a number of "what if" problems incorporated in four and ten period COMPA planning models. Evidence provided by these tests clearly shows that a fully integrated linear programming model can provide information for short term financial planning and decision making.

Extensive analysis of dual evaluators or shadow prices was also undertaken as a part of this study. Experimentation with the COMPA models established that linear programming can be used to trace the opportunity locus of funds in an operating business and identified the impact of stipulated constraints on the slope and range of derived marginal revenue product functions.

This study has produced evidence that clearly indicates the usefulness of linear programming as a tool for short term financial planning. If a representative fully integrated multi-period model of a firm can be developed, it may be used by management to expand both the range and depth of practical short term financial analysis.

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This research was made possible by the complete and enthusiastic cooperation of the two firms that are the anonymous subjects of the field work. I am most grateful for the selfless confidence that the executive groups of both these companies have had in me and the objectives of my study.

This topic and the general methodology that has been used were suggested by Associate Professor Victor L. Andrews who has also been my mentor, friend in court, and caustic critic. I offer my most sincere, but inadequate thanks to him and to Professor John E. Bishop and Associate Professor William L. White who comprised the committee that supervised the preparation of this paper. In addition, I must express my gratitude to many other friends at the Harvard Business School, and particularly Arthur A. Schoenwald, who listened to my ideas and problems and gave so freely of their valuable counsel.

I am also indebted to the United States Navy for making my matriculation possible, to Professor Bertrand Fox and the Division of Research for supporting the computer analysis, and to my family, who bore with me and sustained me.

Boston, Massachusetts
February 1968

Charles W. Young
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Chapter 1

INTRODUCTION

1.1 Purpose

This paper is concerned with the application of mathematical programming to the analysis of short term financial planning problems. However, it will not be an exercise in programming theory or in the construction of abstract mathematical models. Rather, the objective of this work is to establish the extent to which mathematical programming can now be used as a practical management tool for short term financial planning and budgeting. The accomplishment of this objective will require that programming models of operating business concerns be formulated that have the same general properties as the models proposed in the underlying theoretical work. These models will then be used to test, evaluate and qualify the operational usefulness of the theory. In addition, conceptual deficiencies that limit operational applications will indicate areas that need the benefit of new theory or techniques.

1.2 Background

"Mathematical programming" is a term generally used to describe a family of relatively new quantitative procedures that can be useful in solving a variety of business problems. Although the individual programming techniques vary in important characteristics they are all highly efficient methods for the analysis of large amounts of relevant data. Their purpose is to provide decision makers with better, more timely information with the ultimate objective being to improve the quality of management decisions.

A decade ago Henderson and Schlaifer described mathematical programming as follows:

> Mathematical programming is not just an improved way of getting certain jobs done. It is in every sense a new way. It is new in the sense that double-entry bookkeeping was new in the Middle Ages, or that mechanization in the office was new earlier in this century, or that automation in the plant is new today. Because mathematical programming is so new, the gap between the scientist and the businessman—between the researcher and the user—has not yet been bridged. Mathematical programming has made the news, but few businessmen really understand how it can be of use in their own companies.[1]

One of the most versatile and general mathematical programming techniques is "linear programming." Since linear programming was developed by George B. Dantzig in 1947, it has been used extensively by management scientists in attacking complex business problems of many types. Widespread use of linear programming analysis is understandable because theoretically any business situation that can be expressed as a problem in maximizing or minimizing a linear objective function subject to restrictions formulated as linear equations or inequalities can be represented by a linear programming model. As a result the technical and professional business literature abounds with theoretical applications of the technique to business problems.

In addition to theoretical applications mathematical programming also has been profitably employed by many firms in the solution of technical problems. A few years ago the progress of such practical utilization was reported by Rapoport and Drews as follows:

> The growing application of advanced mathematical techniques to the fields of business and economics may very well be one of the most far-reaching developments of the last decade. Whether we are actually on the verge of a revolution in the technology of business administration, as suggested by some forecasters, is still difficult to confirm. But it is a fact that mathematical methods of programming and optimization are increasingly used for the study of industrial and business operations. Like most new 'techniques' mathematical programming has gradually gained acceptance on the basis of a number of special applications. Thus, it is used today as a routine tool for such problems as products blending, setting up refinery running plans, scheduling machine utilization and controlling transportation systems.[2]

This report indicates that the gap between theoretician and businessman has been at least partially bridged. Businessmen are using mathematical programming as a tool for optimizing the operations of

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[1] A. Henderson and R. Schlaifer, "Mathematical Programming: Better Information for Better Decision Making," *Harvard Business Review* (May-June, 1954), p. 73.

[2] L. A. Rapoport and W. P. Drews, "Mathematical Approach to Long-Range Planning," *Harvard Business Review* (May-June 1962), p. 75.

segments of their firms. However, one defect in current applications is the failure to use fully integrated models incorporating financial constraints. The models that are used may fail to recognize all possible constraints and, in the context of the firm, decisions are implicitly suboptimizing if all limitations and interdependencies are not explicitly considered. In particular, short-term optimization of the returns to the firm depends on how financial resources are divided among competing possible uses. Beranek recognizes this situation and comments as follows:

> When a given amount of resources are allocated to a departmental production manager, he determines his optimal rate of output by the process of suboptimization. But determining the amount of resources which should be made available to a department, and especially in the face of over-all firm financial constraints—e.g., existing cash balance, available trade credit and limited external sources of money, and balance sheet constraints like a minimum current or acid test ratio—is a financial problem, a problem which usually cannot be suboptimized with satisfactory results.[3]

The potential managerial usefulness of fully integrated models has recently been made apparent. Several management scientists have demonstrated that financial limitations and interrelationships are susceptible to formulation as linear programming constraints. In addition they have shown how a fully integrated model can provide valuable financial information that would be difficult or impossible to obtain otherwise.

One of the first and best theoretical applications of mathematical programming to the optimal allocation of funds within an enterprise was developed by Charnes, Cooper and Miller.[4] Their work is important because it not only forms the theoretical foundation for most of the later work in the area but it also clearly illustrates the potential managerial usefulness of the financial information provided by both the primal and dual solutions. They demonstrated that linear programming can be used as a technique for simultaneous solution of the related problems that must be solved before a firm's resources can be rationally deployed. These problems include: (a) determining the optimal joint operating and financial plan over the relevant planning interval; (b) establishing the "yield" to the firm

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[3] W. Beranek, *Analysis for Financial Decisions* (Homewood, Illinois; Richard D. Irwin, Inc., 1963), p. 81.

[4] A. Charnes, W. W. Cooper and H. M. Miller, "Application of Linear Programming to Financial Budgeting and the Costing of Funds," *Journal of Business* (January 1959).

of each of the various possible changes in its asset structure; and, (c) finding the opportunity cost of funds in the firm, in the sense of the prospective rate of return on an increment of funds committed to the enterprise. Their analysis also proves that the dual evaluators or shadow prices produced by including financial constraints in the linear programming model are the opportunity costs needed for optimal allocation of funds. The practical value of the shadow prices is then demonstrated by using them in simple examples to evaluate investment and financing plans.

The Charnes, Cooper and Miller discussion also strongly implies the usefulness of linear programming in the analysis of financial strategies. They explicitly show how alternative financing assumptions can be incorporated by adding various forms of financial constraints to their simple warehouse model. Solution of these augmented models will then permit management to simulate a number of financing situations in order to compare the profit, cash flow and operating implications of each plan.

Beranek[5] has also illustrated how financial constraints and alternative sources of financing can be included in a linear programming model that will be useful in planning the interrelated activities of a firm over a number of periods. His work emphasizes the value of the information provided by the financial dual evaluators and the usefulness of linear programming for sensitivity analysis in financial planning. In addition he extends the multi-period model with financial constraints representing balance sheet ratios and other policy considerations.

In both the Beranek and Charnes, Cooper and Miller applications the usefulness for financial planning attributed to linear programming derives from the use of fully integrated multi-period models. That is, constraints representing financial limitations have been combined with sales, production or other operating constraints and objective functions have been expanded to include recognition of financial income and expense. Incorporating the period-by-period cash flow characteristics of each operating variable in the financial constraints forces the model to recognize that the variables absorb and throw-off cash at different rates. Then by continuous evaluation of profit opportunities the logic

[5] Beranek, *op. cit.*, Chapters 13 and 14.

of the method rations limited funds among products and across periods by seeking the balance between unit profit and working capital turnover that will optimize the specified objective. This evaluation and trading process finds the next largest increment to the objective that can be obtained by altering the values of the operating variables. When it is no longer possible for the model to improve its position the final values of the variables constitute an optimal plan and the dual evaluators are the marginal values to the model of unit changes in each resource. If the resource is cash its shadow price is the opportunity cost of funds in the firm.

Another related application of linear programming to financial planning has been proposed by Ijiri, Levy and Lyon.[6] They have developed a single period fully integrated model that combines linear programming with double-entry accounting in a unified approach to total enterprise planning. Their model is used to bring the firm into the best possible end-of-period balance sheet position and to supply information to management that will indicate the dollar consequences to be expected from altering the firm's policies or environment. This application again indicates the flexibility of linear programming for planning purposes and shows that problems can be formulated in terms of any reasonable set of variables, including the debits and credits of double-entry accounting.

Some of the most recent work in the application of linear programming to financial problems has been done by Robichek, Teichroew and Jones.[7] They have demonstrated that by taking advantage of mathematical programming's unique ability to determine the optimal "package" from a set of alternatives, none of which is superior in every relevant characteristic, it can be used to find the least cost financing plan, given a cash budget for a finite number of future periods and a clearly defined array of sources. However, their model is strictly financial and does not represent a fully integrated approach to enterprise planning.

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[6] Y. Ijiri, F. K. Levy and R. C. Lyon, "A Linear Programming Model for Budgeting and Financial Planning," *Journal of Accounting Research* (Autumn 1963).

[7] A. A. Robichek, D. Teichroew and J. M. Jones, "Optimal Short Term Financing Decisions," *Management Science,* (September 1965).

There is no evidence, however, that any of the theoretical work cited has been validated by application to operating businesses. In fact, the only direct application of linear programming to any type of financial problem that has been found is a 1401/1440 program developed by IBM for savings and loan associations. This program, titled "Program for the Allocation of Resources for Savings and Loan Associations" (PARSAL)[8], is designed to assist savings and loan management in making decisions which determine profitable levels for the association's assets and liabilities. PARSAL translates input data that describe the current financial status of the association, legal requirements it must meet, and its individual policies into linear constraint equations that are then solved by linear programming to maximize profit for the planning period. It is a fully integrated model in the context of savings and loan associations and has a few dynamic features. However, it is essentially a one period analysis. The length of this period can be specified by the program's user to fit its individual situation and the problem is updated and rerun to simulate additional periods.

Although a program such as PARSAL is available for a very specialized and structured application there is no indication that linear programming is being used to construct models of individual firms that can be used to attack specific financial problems. In addition, it has not been established that the detailed financial information required to formulate a fully integrated model is readily obtainable in the average firm or if the existing theory can be adapted to available data. Therefore, it seems reasonable to conclude that the practicality and usefulness of linear programming as a tool for financial management has not been tested.

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[8] International Business Machines Corporation, *1401/1440 Program for the Allocation of Resources for Savings and Loan Associations* (1401-FB-14X and 1440-FB-13X), International Business Machines Corporation, Data Processing Division, White Plains, New York.

Chapter 2

THE PLAN OF THE STUDY

2.1 Application of the Theory

As mentioned in the preceding chapter, the purpose of this work is to determine if fully integrated, multi-period, linear programming models can be useful to management in the optimal allocation of financial resources within the firm. Because this is a test of "applicability" no attempt will be made to develop new theory. Rather, existing programming theory and techniques will be adapted and used to construct specific models that are as representative of individual real firms as possible.

The main thrust of this work will be to determine if the information required to formulate financial constraints in the linear programming models can be obtained and if theoretical constructs can be adapted to represent the financial characteristics of operating businesses. As pointed out in the theory the value of the information provided by a model derives from its ability to optimally use the data furnished as input.

The representativeness of the model depends to a large extent on the availability of parameter values that reflect the average experience of the firm being studied. The hypothetical and deterministic examples used to illustrate theoretical applications assume all essential data are provided and little attention is given to their source or expected availability in operating firms. In the operating firm input data for the model can only be obtained by evaluating and summarizing the accounting and statistical information that can be found.

The model of a firm being studied will be formulated from the point of view of the businessman and ultimate user of the information that it may provide. In this regard the object of the application

will be to make the model as useful as possible to management for financial planning and decision making. This approach also responds to a challenge issued by Henderson and Schlaifer, "The fate of mathematical programming, in other words, lies today in management's hands. The scientists, the inventors, have done their work; it is now up to the users."[1]

Mathematical programming is a management tool, not a substitute for decision making. Therefore, if its usefulness for financial planning is to be objectively evaluated it is necessary to define expected benefits. As indicated in the review of theoretical applications, mathematical programming should provide management with two basic categories of information required for rational allocation of resources within the firm. The first is an integrated, multi-period plan for production, marketing and finance that will yield the largest prospective returns. Because the linear programming analysis evaluates the dissimilar cash consumption characteristics of each operating variable in conjunction with all interdependencies, limitations and opportunities specified in the model, this plan will ration available funds in the best possible way. The plan will also indicate the amount of financing required in each period to support the prescribed level of operations, the periods in which loan balances should be reduced and the periods in which cash will be in excess supply.

The second category of information expected is the marginal value to the firm of changes in the asset structure and the shadow price or opportunity cost of funds in each period. The shadow prices of all resources, including funds, can then be used to develop optimal plans for conversion of funds into physical capacity or other resources and as hurdle rates in capital budgeting analysis. Inter-period differences in shadow prices can be exploited to determine the value of deferred versus immediate payment and the most profitable schedule for borrowing and lending. Henderson and Schlaifer emphasize the value of the shadow price information.

> Determination of the most profitable program under a particular set of circumstances is by no means the only advantage which management can derive from the intelligent application of mathematical programming. In many situations the technique will be of equal or

[1] Henderson and Schlaifer, *op. cit.*, p. 94.

> even greater value as the only practical way of obtaining certain cost and profit information that is essential for sound decisions on both short-run and long-run problems of many kinds.[2]

In summary, the mathematical programming analysis can be expected to provide two general types of information. First, optimal operating plans and second, the marginal values of additions to assets. However, it must be remembered that this information is obtained from a mathematical model of the firm under analysis. This model is obviously an abstraction of reality and the optimality of problem solutions obtained through its use are only with respect to the model. Dorfman, Samuelson and Solow emphasize this point:

> The linear-programming models we shall develop will, of course, not be strictly accurate representations of the economic situations with which they deal. Strict descriptive faithfulness is an unreasonable demand to make of any conceptualization.[3]

Conceding the approximateness of the information provided by the programming analysis, how can this information be best used by management in financial decision making? Henderson and Schlaifer[4] consider that mathematical programming is of greater value for planning than in the making of immediate operating decisions. Rapoport and Drews deem the functions of mathematical programming to be:

1. To evaluate planning factors and alternatives as they arise, rather than to prescribe courses of action.

2. To provide the planner with systematic, more powerful means of analysis, rather than to reduce his responsibility by the use of automation.

3. To help management explore policies and objectives in greater depth, but not to introduce a substitute for decision making.[5]

Dorfman, Samuelson and Solow[6] point out that a conceptual model should grasp the strategic relationships that control the phenomenon it describes and that it thereby permits us to manipulate, i.e., think about the situation. This is essentially the framework within which the proposed evaluation

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[2] *Ibid.*, p. 90.

[3] R. Dorfman, P. A. Samuelson and R. M. Solow, *Linear Programming and Economic Analysis* (New York: McGraw-Hill Book Company, 1958), p. 9.

[4] Henderson and Schlaifer, *op. cit.*, p. 94.

[5] Rapoport and Drews, *op. cit.*, p. 89.

[6] Dorfman, Samuelson and Solow, *op. cit.*, p. 9.

of mathematical programming as a tool for short term financial planning will be conducted. That is, does mathematical programming provide a useful and practical way for thinking about financial plans and problems? Can mathematical programming models of a firm be developed that will predict the funds needs of various possible operating plans, the opportunity costs of alternative financial strategies and the best attainable objective over a range of conditions?

2.2 Characteristics of the Model

In this work only the general linear programming model will be used. That is, the model will seek to maximize the function,

$$z = \sum_{j=1}^{N} c_j X_j$$

subject to the restrictions,

$$\sum_{j=1}^{N} a_{ij} X_j = b_i \qquad i = 1, 2, \ldots, m$$

$$X_j \geqslant 0$$

where: X_j = j^{th} decision variable in the solution vector

c_j = a given constant representing the cost or revenue of including one unit of X_j in the solution vector

b_i = capacity of the i^{th} process or available quantity of the i^{th} resource

a_{ij} = a given constant representing the quantity of the i^{th} resource consumed when one unit of the j^{th} variable is included in the solution vector

Although there are other specific model forms available in the mathematical programming family, linear programming is the most widely used and understood of the techniques. Also, it is readily adaptable to a variety of approximately linear situations and has the most efficient and readily available computer codes.

One objection to the use of linear programming is that economic relationships are often non-linear and production and cost functions are usually thought of as curves. Linear programming analysis

requires linear functions, fixed input and output prices, and constant returns to scale. Therefore, application of linear programming to most situations requires that obviously non-linear functions be approximated by linear functions. This can be done without significant loss of precision if the ranges of values within which the approximations are to be applied are selected with care. However, as with standard "break-even" analysis the extrapolation of a linear function beyond its relevant range is a dangerous procedure. Beranek justifies the use of linear cost functions with the following reasoning:

> Linear cost functions are useful concepts. In addition to simplicity, there are other reasons which justify their study. First, of the totality of cost relations which confront the typical decision maker, many are in fact, linear. Secondly, linear expressions often serve as useful approximations to more complicated, more cumbersome expressions. Even though we know that a cost relationship is non-linear, the exact expression may be difficult to obtain and both costly and time consuming to use in decision making. A straight-line approximation may serve the purpose of the analysis. We know that it is in error, that it is not the true relationship, but this is not sufficient grounds for rejecting it. The relevant consideration is the magnitude of the error and its impact on the *size of the mistake to be made* in the ultimate financial decision. If, as a consequence of the approximation, the costs involved in making a mistake are trivial, the linear estimate may be justified.[7]

The assumption of fixed input and output prices appears to correspond to the situation faced by many firms today. Few business concerns can influence the costs of labor or raw material inputs and most firms are subject to prices for their output that are competitively determined, made rigid by oligopoly, or regulated by government. This assumption is made even less objectionable because the applications of linear programming to financial management in this work will be limited to short term planning periods during which stable prices can be expected. However, a multi-period model can recognize inter-period changes in costs and prices.

The linear programming financial models should also make allowance for the time value of funds. Discounting and present value concepts have become an integral part of the theory of financial management and it is certainly well known that future flows should be evaluated in terms of applicable opportunity costs. However, for several reasons, the time value of funds will not be explicitly recognized in this study. First, the technique used to include discounting in linear programming models is well known

[7] Beranek, *op. cit.*, pp. 44-45.

and not central to any analysis planned.[8] Second, all planning periods represented by the models to be used in this study will be for one year or less and the added precision to be gained by discounting flows over the very short term does not justify including discount rates in the models. Finally, because fundamental, intra-firm financial relationships determine the opportunity costs of funds, the masking effects of arbitrarily assigned discount rates would make it more difficult to understand and interpret the empirical work.

Another problem in the application of linear programming to "real world" situations is how to effectively deal with uncertainty. A practical technique for including the effects of uncertainty in the analysis has not been developed and most theoretical work assumes a world of certainty. However, when only the immediate future is involved, as in short term planning, it is felt that uncertainty is somewhat constrained. In general, all beginning balances are known and costs, prices and production capacity are fixed. The pattern of cash receipts is established by credit terms and can be approximated by analysis of collections as a function of previous months' sales. The same general conditions apply for payment of trade accounts. Availability of lines of credit and applicable interest rates are also usually negotiated in advance of need. This leaves demand as the only parameter subject to significant uncertainty. Most management scientists either ignore this uncertainty entirely or mention that uncertain parameters can be replaced by their expected values. This is the procedure advocated by Robichek, Teichroew and Jones,

> The LP model under certainty, as developed so far, can be regarded as maximizing expected value for a probabilistic model in which the distribution of parameters and requirements are replaced by their expected values.[9]

Another approach which is sometimes advocated involves evaluating several sets of possible demands by submitting them to the programming model.[10] This simulation approach allows management to

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[8] See, for example, Beranek, *op. cit.*, p. 448.
[9] Robichek, Teichroew, and Jones, *op. cit.*, p. 31.
[10] Rapoport and Drews, *op. cit.*, p. 85.

review an array of solutions and note the sensitivity of decision variables and profit to changes in levels of demand and product mix.

Possibly the best technique for incorporating uncertain demand forecasts in the analysis is the linear decision rule model, for scheduling aggregate work force, production and inventories, developed by Holt, Modigliani, Muth and Simon.[11] Because their rule is derived from a quadratic cost function the only information about future sales that enters into the optimal decision rule is the expected value of sales in each relevant future period. The same rule that yields the best decision in the presence of certain future conditions also yields the best decision when actual sales figures are replaced with expected future sales figures.[12] However, limitations on decisions can only be introduced into the model through the cost function by adding terms that become very large as resource limits are approached. These pseudo-costs are not an effective way to introduce constraints and the linear decision rule cannot normally be used directly where resources are limited. In fact, the inventors of the technique advise that alternative mathematical models, e.g., linear programming, be investigated where it is necessary to handle significant bottleneck problems.[13] For this reason it does not appear that the linear decision rule model is applicable in situations where limited financial resources are the problem and the basis for the study.

In this research most models will be based on historical demand and can be characterised as assuming that demand is known with certainty. Manipulation of the model to simulate its use for financial planning may involve parametrically varying historical demand to determine the impact of of changes in sales on funds requirements and opportunity costs. However, if the need arises to use future demand or other uncertain parameters in the models they will normally be represented by their expected values.

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[11] C. C. Holt, F. Modigliani, J. F. Muth and H. A. Simon, *Planning Production, Inventories, and Work Force* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1960).

[12] *Ibid.*, p. 123.

[13] *Ibid.*, p. 117.

2.3 Selecting Firms for the Tests:

Most of the work that has been mentioned used simple examples to illustrate the application of linear programming to the solution of financial problems. While such applications facilitate presentation of theoretical concepts and analysis of informational output they do not touch on the difficulties to be expected in the formulation of models that represent real firms.

Because the proposed research is primarily concerned with applications of linear programming to short term financial planning in operating businesses the type of firm that seems most likely to benefit from the study must be specified. First, a company must have certain characteristics before a linear programming model is feasible. That is, it must have established factor costs and product prices. Production should consist primarily of standard products that are made to inventory and there should be approximately constant returns to scale in the normal operating range. In addition, a firm must have established and/or predictable funds flow characteristics. Second, the company must be subject to some form of financial limitation. If cash is redundant there is no basis for the study.

The selection of firms to participate in this study has also been limited by practical considerations. First, because this research is a one-man effort with limited financial support, it was necessary to concentrate on firms in the Boston area. Second, only firms that offered complete cooperation could be used. Third, the limited amount of time available required that consideration be limited to small or medium-size companies.

In the search for companies to use as bases for linear programming models, the first and second conditions were easily fulfilled. Because Greater Boston is heavily industrialized it was relatively simple to select a number of firms that appeared to be acceptable subjects. The executives of all firms contacted were most cooperative and generous with their time, and in general were completely willing to participate in this study. However, even very small firms are often complex, particularly when sophisticated technology or manufacturing is involved, and other firms either do not have set prices for their products or they use raw materials that have unpredictable costs.

These "problems" that complicated selection of firms to participate in this study do not necessarily indicate that there are few potential applications of linear programming for financial planning. Rather the problem was more the restrictiveness of the prerequisites. Both larger and more complicated firms could have been included in the study if a research team had been available to gather data and assist with analytical chores. In addition, the absence of a binding cash shortage does not nullify the potential usefulness of a linear programming model for evaluating plans and budgets. In fact, realistic representation of an absolutely binding cash position may reduce model flexibility to the point that it cannot be used to evaluate markedly different strategies.

After a few firms that appeared to have all the necessary characteristics had been identified it was decided that fully integrated models would be formulated to represent two companies. Although the results obtained from testing the model of one firm should provide considerable information about the usefulness of linear programming, overall findings will have additional validity if at least two different types of firms are studied. The two companies that were selected and asked to participate in this study readily agreed and have cooperated in every possible way. However, because undisguised data is used, both of these concerns wish to remain anonymous. One company is a medium-size manufacturer of confections and will be called "Company A" or COMPA. The other firm is a retail mailorder warehouse and will be referred to as "Company B" or COMPB.

2.4 Outline of the Tests

The tests that will be made using COMPA and COMPB can be divided into three parts. The first tests are planned to accomplish one of the important objectives of the research. That is, they will attempt to determine the extent to which accounting and other information that is routinely accumulated in an average firm can be used to make estimates of the interrelationships among operating and financial functions that are adequate to permit formulation of a representative linear programming

model. In this initial phase the company's organization and operations will be studied to identify those characteristics and interdependencies that are the most important determinants of profitability and cash flow. Detailed data concerning these basic properties must then be gathered and analyzed to ascertain parameter values that can be used to transform fundamental relationships into constraint equations. If available data can be used for this purpose then it will be possible to formulate a fully integrated model of the company. If not, data deficiencies must be explained and a determination made concerning how, or if, they can be overcome.

The second part of the tests will find out if the fully integrated model of each company, assembled from the constraints, variables and parameters determined in phase one, represents the subject firm. The "representativeness" of this model will be evaluated by simulating a planning period, consisting of several sub-periods (months), from the company's recent history. Simulation will be forced by requiring the model to produce and sell the same mix and volume produced and sold during the period being simulated. If profit, factor consumptions, cash flows, inventories, receivables and payables balances, and demands for external funds are approximately the same as those that actually occurred then it will be concluded that, within the normal operating range, the model is representative and can be used for planning.

The third test phase will use the representative model developed in part two to analyze a series of "what if" questions. A hypothetical alternative will be defined and the model's parameters modified as necessary to incorporate it. The solution to each of these problems should indicate the effects on profit, cash flow, demand for external funds, etc., that would have occurred if the firm had followed the hypothetical course of action. In addition, parametric variations will be used with several of these problems to provide a schedule of the opportunity costs of funds at various levels of availability. Although a number of "what if" problems will be used, limited research and computer time preclude more than a cursory examination of the many and varied types of financial planning and budgeting problems that might be evaluated.

If the information obtained from these "what if" problems appears to be realistic, and if the differences in the optimal decisions required to implement various strategies are clearly indicated, the linear programming model will have been shown to be useful in the evaluation of alternative courses of action and in defining the optimal set of decisions required to implement them. In addition, if meaningful shadow prices for funds can be determined the tests will have established that linear programming can be used to find the opportunity costs of funds in operating firms.

The series of tests outlined above will be considered successful if they clearly indicate the extent to which linear programming can be used by businessmen for short-term financial planning. Reasonably useful applications in both companies will establish that linear programming is a potentially valuable financial management tool. Less useful results may suggest improvements in theory and/or technique that are needed. Hopefully, the nature and extent of needed improvements can be defined by analysis of obvious deficiencies in the test models. If lack of usable data is an obstacle, it may also be possible to specify plans for data accumulation that will enable more useful models to be built in the future after needed information becomes available.

Once a programming model is developed it is only reasonable to expect that it will be usable, with periodic maintenance, for a considerable period of time. This requirement is of considerable importance in determining the ultimate usefulness of the model. Not only are models usually difficult and time consuming to construct, they also should be readily available for use in planning and evaluation studies. Routine updating should require no more than incorporation of known changes in operating conditions and adjustment of the initial and terminal periods to match the problem under consideration.

Chapter 3

THE FULLY INTEGRATED MULTI-PERIOD MODEL

This chapter is intended as a bridge between the theoretical applications of linear programming that are being investigated and the applications to operating businesses that comprise the tests. For this reason the general properties of the fully integrated model and the practical aspects of model building, data requirements definition, and constraint formulation will be discussed before proceeding to the field work. In the remainder of this paper all descriptions of linear programming formulations assume an acquaintance with the technique. However, if more information is desired on the technical aspects of linear programming, the reader is referred to the many general texts that are available.[1]

3.1 General Properties of the Fully Integrated Model

The power of the fully integrated model to optimize the overall operations of the firm derives directly from the inclusion of constraints that recognize all important production, marketing, financial and policy interrelationships. Recognition of all meaningful costs and opportunities permits the model maximum flexibility to trade-off between traditionally separate functional areas. When the fully integrated model is expanded to include more than one period, opportunties are also provided for optimal distribution of production, sales, and cash flows over time.

To improve its usefulness, a mathematical model should always be as strictly representative of the subject with which it deals as practicable. In abstraction this criterion can be construed as requiring

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[1] For an elementary discussion see: Harvey C. Bunke, *Linear Programming: A Primer* (Iowa City: State University of Iowa, 1960); Beranek, *op. cit.*, Chapter 13, or William J. Baumol, *Economic Theory and Operations Analysis*, Second Ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), Chapter 5. For a more comprehensive text see: Robert W. Llewellyn, *Linear Programming* (New York: Holt, Rinehart and Winston, Inc., 1960.)

the inclusion of constraints and variables to quantify every conceivable relationship within the firm. However, the rapid growth of the linear programming matrix in the multi-period model militates against this course of action and demands that only fundamental considerations be included.

Representativeness is also dependent on availability of high quality data and each coefficient must be precisely quantified to insure that marginal decisions are rational. In addition, a model must be formulated on a homogeneous base. That is, all dissimilar data elements must be converted to a common denominator or individual coefficients must include conversion factors to relate unlike variables. For example, several COMPA production capacity constraints reflect in-process relationships and both value and weight are added subsequently. Net weight of each product at various stages of completion must then be converted to finished weight, the stipulated common base, in order to maintain homogeneity. This is done by using the ratio, end of process weight to finished weight, to modify the coefficient that reflects the input-output relationship for each product.

The typical fully integrated multi-period model of a firm includes five distinct sections. The first of these sections contains constraints that specify the production capability and input-output restrictions within which the firm must operate. This section also includes the multi-period production, sales, and inventory tie equation set. The tie-equations make explicit the requirement that in every period beginning inventory, plus receipts, less cost of goods sold must equal ending inventory and thus cement the multi-period model together.

The second section of the model consists of equations to accumulate inputs required in the financial constraints. If total sales, monthly shipments, etc., must be recognized in the cash constraint, formulation is simplified by adding individual products and/or periods in accumulation equations. Cumulant variables that reflect total values determined by these equations are then entered directly in the cash constraint. Technically, this section could be omitted. However, the use of equations to accumulate input values provides flexibility, makes the operation of the model more easily understood, and facilitates planning or other revisions.

Financial and policy constraints comprise the third section of the model. The cash flow constraint set is the heart of this section and its inclusion provides the model with its potential usefulness for financial planning purposes. In most situations the addition of a cash flow equation is the major extension necessary to convert a sub-optimizing model to the fully integrated form. Cash flow constraints are similar to the inventory investment equations in that they enforce the fundamental identity that the beginning cash balance, plus receipts, less expenditures must equal the ending cash balance. In addition, the cash flow constraint is supported by other equations that limit and regulate the availability and/or flow of external funds and enforce financial policy.

The fourth section of the typical model includes the equations and inequalities that are used to incorporate the parameters within which it must operate. When the fully integrated model is tested by requiring it to simulate a period in the firm's history it is necessary to provide the model with actual production, sales, inventory levels, manhours worked, quantity of raw materials consumed, etc. The model must then optimize within these parameters. When the fully integrated multi-period model is used for planning purposes, the incorporation of assumptions is accomplished to a large extent by varying parameters or stipulated right hand side values of constraints in this section.

The objective function or functional is considered to be the fifth section of the model. The functional is formulated to reflect management's operating objective. This objective may be to maximize some measure, e.g. profit or revenue, or to minimize cost. The functional controls the substitution of variables in the solution process and serves to drive the solution to optimality with reference to the specified objective. Generally, terms in the functional accumulate explicit costs and revenue elements that have counterparts in the operating statement. However, implicit costs are also included to establish priorities or penalities that encourage or discourage various courses of action. The functional provides a flexible tool and sophisticated use of this tool can greatly enhance the usefulness of the fully integrated multi-period model for planning purposes.

Before proceeding to discuss the various elements of the fully integrated multi-period model in more detail, it should be noted that the models that will be developed in this paper will generally include the following four basic simplifying assumptions that are usually a part of any theoretical work concerned with optimization of short term financial decisions.[2]

1. The minimum cash balance required at all times is specified with reference to solvency or some other overriding consideration.
2. The pattern of cash inflows and outflows or the functional relationship of such flows to operating variables is completely known.
3. The cash flows are partly or wholly discrete. This is also the assumption of discrete time periods within which the time distribution of flows makes no difference.
4. Explicit costs for each external source of funds are known.

3.2 Identification of Decision Variables

In most non-financial businesses the basic operating decision concerns which individual products to make or buy and when and in what quantity each of these products should be produced or purchased. The decision to include an item in the firm's product line and to offer it for sale necessarily requires additional decisions concerning levels of inventory and marketing policy. If the fully integrated multi-period model is to make these same decisions it must include variables to reflect the production, sales and ending investory balance of each product in every period.

The accumulation section of the model requires that variables also be defined to represent totals or cumulants. These variables serve an internal, technical function and seldom enter directly into the decision making process, i.e. acquire a cost or benefit coefficient in the functional. The number and specific definitions of these variables is determined by the nature of the model and the needs of the model builder.

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[2] Robichek, Teichroew, and Jones, *op. cit.*, p. 7.

The third type of variable is peculiar to the fully integrated formulation of the linear programming model. These are the financial decision variables. When cash flow and other financial constraints are added to the basic model it is necessary to include variables to accumulate the amount of funds borrowed from each source, the quantities of excess cash loaned at interest and the end of period cash balance. In some formulations variables are also required to reflect cash shortages or unsatisfied fund requirements created by the hypothetical attainment of a desirable objective. Although some variables in this group may be passive, i.e. not assigned specific cost or revenue coefficients in the functional, variables that reflect financial balances are normally active in the decision making process.

3.3 Selection of a Homogeneous Base

After a set of decision variables has been defined it is necessary to determine a unit of measure in which it can be expressed. Financial variables must always be quantified in dollars or other appropriate monetary units. However, variables that represent individual products should be expressed in the most useful and meaningful terms. For example, a mix of physically homogeneous products lends itself to a common unit base. While these products are dissimilar in many aspects, they can all be measured in tons, pounds, feet, etc. Conversely, a heterogeneous product mix can usually be expressed only in monetary terms. To determine if it is more profitable to produce a ton of cement or a tractor the model must be able to compare their relative merits and a common denominator is required.

Once a base is selected it must be used consistently throughout the model. If the base is units, then units produced and sold must be comparable and all decisions within the model must be in terms of this measure. In addition, if the solution values determined by the model are to be useful they must be expressed in commonly accepted terms. Management is not accustomed to discussing sales estimates expressed as direct cost of sales or purchases at their resale value. To insure that solutions can be readily interpreted, purchases should be extended at net cost, sales at selling price, and inventory at normal carrying cost.

3.4 Aggregation of Products

The size of the linear programming matrix is directly attributable to the number of product related decision variables included in the model. Although a representative model must be permitted to consider all production and sales opportunities, ease of understanding, usefulness of solution results, and compatability with available linear programming computer codes requires that products be analyzed and grouped so that only variables with significantly different characteristics are included in the model. In many firms that produce a limited number of products it is possible to combine those that have similar physical properties, production costs and selling prices. This was found to be the situation in COMPA where initial intuitive aggregation was used to reduce the number of products that had to be considered from over 100 to 46. However, further aggregation of COMPA's products required the use of analytical procedures to determine weighted average direct unit cost, price, margin and factor consumption coefficients. The success of such an averaging procedure can usually be tested only by using the aggregated products in a model that simulates actual experience.

If the product line to be considered is both large and diverse it is normally only practicable to aggregate by financial characteristics, e.g. percentage markup, percentage cost, etc. This is the situation that typically exists in a wholesale or retail merchandising firm with a broad product line. Except for those handling basic commodities, the typical firm that could be analyzed in the context of the theoretical "warehouse model" would be in this category. Experience gained with COMPB indicates that aggregation of products in a merchandising firm, while an essential prerequisite of model formulation, is extremely difficult and prone to fail. Because only one or two product characteristics can be explicitly recognized and because product sub-groups must be broad, any averaging procedure tends to provide a suspect mean with a large variance. In addition, when means or expected values are not markedly different, all product groups appear to be equivalent and there is no basis for the model to discriminate among them.

3.5 Non-Financial Constraints

The non-financial constraints form the base for the fully integrated model. In large part they are the same constraints that would be found in a sub-optimizing model used to schedule production. If a firm were using linear programming for production planning, most of the constraints from that model should be directly transferrable to this part of the fully integrated model.

a. General Input-Output Inequalities

Because the model's primary function is to allocate resources, a constraint set must be specified for each limited physical resource. These inequalities make explicit the input-output relationships that exist in the production process. In addition, they establish an upper limit on the amount of each resource that is available in every period. In the multi-period model the input-output constraints have the following form:

$$\sum_{j=1}^{N} a_{ij}P_{jT} \leqslant B_{iT} \qquad \begin{matrix} i = 1, \ldots, X \\ T = 1, \ldots, M \end{matrix} \qquad \text{(3-a)}$$

where:

P_{jT} = Total production of product j, period T.

a_{ij} = Coefficient to reflect the amount of capacity i consumed in the production of one unit of product j.

B_{iT} = Maximum amount of resource i available in period T.

i = Individual limited resource, from 1 to X.

j = Individual product, from 1 to N.

T = Individual time period, within the planning horizon, from 1 to M.

In these constraints the decision variables are the output of each product j, and the right hand side value B_{iT} is the maximum amount of each resource available in every period. To make these constraints specific it is also obviously necessary to state quantitatively the coefficients a_{ij}. Most of the specific production and direct cost data normally needed for this purpose should be provided by a good

standard cost system. However, if this information is not available, as was the case with COMPA, it will have to be developed before the constraints can be made operational.

b. The Tie or Inventory Investment Equations

This set of constraints is of critical importance in the multi-period model because it serves to tie each product's end of period inventory balance to production and sales for the same period. That is, it insures that in every period, beginning inventory, plus production, less sales equals ending inventory. In the warehousing model these equations serve essentially the same purpose but, because manufacturing is not involved, they must also incorporate most operational or physical limitations.

The tie equations can be formulated in either of two equivalent ways. The cumulative form:

$$I_{j0} + \sum_{t=1}^{T} P_{jt} - \sum_{t=1}^{T} S_{jt} = I_{jT} \qquad (3\text{-b, } 1)$$

or

$$-\sum_{t=1}^{T} P_{jt} + \sum_{t=1}^{T} S_{jt} + I_{jt} = I_{j0}$$

$$j = 1, \ldots, N \qquad T = 1, \ldots, M$$

or the simpler non-cumulative form:

$$I_{jT-1} + P_{jT} - S_{jT} = I_{jT} \qquad (3\text{-b, } 2)$$

or

$$-P_{jT} + S_{jT} + I_{jT} - I_{jT-1} = 0$$

$$j = 1, \ldots, N \qquad T = 1, \ldots, M$$

where:

I_{j0} = Inventory balance of product j, end of period 0. (beginning inventory)

I_{jT-1} = Inventory balance of product j, end of period T-1. (if $T = 1$, $I_{j0} = I_{jT-1}$)

I_{jT} = Inventory balance of product j, end of period T. (ending inventory)

P_{jt} = Total production of product j, period t.

S_{jt} = Total sales of product j, period t.

i,j,T = See section 3.4a.

The decision to use the cumulative or non-cumulative form of this constraint has no effect on the primal solution. However, in the dual the shadow price of each row in the non-cumulative form becomes the sum, from T to M, of the individual shadow prices that are provided by the same constraints in cumulative form. If the variables P, S, and I are not expressed in homogeneous units, coefficients must be used to convert them to a common base. For example, if 'P' variables are defined in terms of net cost and 'S' variables in terms of selling price it will be necessary to convert one or both of them to provide a common unit of measure in order that internal consistency can be maintained within the equation set.

3.6 Financial Constraints

The addition of financial and policy constraints to the basic production and/or inventory investment model is the major extension necessary to convert it to the fully integrated form. However, the financial constraints must be precisely formulated if the cash flow, borrowing, lending and financial policy situation that exists in the firm is to be adequately represented in the model.

a. The Accumulation Equations

Accumulation equations are useful in a large and complex fully integrated model to provide aggregate inputs for the cash flow and other financial constraints. Although not required, these equations greatly simplify formulation of the cash flow constraint. For example, to reflect total sales revenue for period T in the cash flow constraint it would normally be necessary to include a term for each S_{jT} variable. However, by inclusion of a set of equations to provide a total sales input, only one

term is needed in the cash flow constraint. Eliminating individual variables from the cash flow equation that may have a number of cash inflows and/or outflows associated with each of them also sharply reduces the number of complex summary coefficients that must be developed and interpreted.

The accumulation equations are generally simple and have approximately the following form:

$$\sum_{j=1}^{N} Q_{jT} - SUM_{QT} = 0 \qquad (3\text{-c}, 1)$$

$$T = 1, \ldots, M$$

or

$$\sum_{j=1}^{N} q_j Q_{jT} - SUM_{QT} = 0 \qquad (3\text{-c}, 2)$$

$$T = 1, \ldots, M$$

where:

Q_{jT} = Any decision variable, e.g. P, S, I, etc., related to product j, period T.

SUM_{QT} = Sum of the specified decision variable Q across all products, period T.

q_j = Optional coefficient to convert the unit measure of decision variable Q. For example, if Q_j in the solution vector is expressed in units, and SUM_{QT} is required to be in pounds, q_j would be the weight of one unit of product j in pounds.

b. The Cash Flow Constraint

The cash flow constraint is the heart of the fully integrated model. It recognizes the cash absorption or throw off of each variable, determines net cash needs, selects the least cost source of external funds and applies available funds to the most profitable opportunities. However, its primary structural purpose is to establish that, in each period, the excess of cash outflows over inflows cannot exceed the beginning cash balance. In many of the theoretical discussions that have been cited this constraint is formulated as an inequality and a positive end of period cash balance would appear in the solution vector as a slack variable. However, in the models that will be formulated in later sections of this paper the cash flow constraint is changed to an equation and a real variable is added to reflect any positive end of period cash balance.

The cash flow equation is not rigidly defined because maximum flexibility is required to insure that it represents a particular firm's cash consumption and generation characteristics. However, the typical general cumulative form is:

$$\sum_{t=1}^{T} \sum_{Q=Q_1}^{Q_n} SUM_{Qt} - \sum_{t=1}^{T} EXTFUNDS_t + ECBAL = BCBAL + \sum_{t=1}^{T} NMR_t \qquad (3\text{-d})$$

$$T = 1, \ldots, M$$

where:

SUM_{QT} = See 3.5a. The double summation term in this equation represents the cumulative net cash flow, from period 1 to T, generated as a result of all production, sales and inventory decisions. Coefficients may also be added to these terms to incorporate lags in receipt or disbursement of cash.

$EXTFUNDS_t$ = The sum of all external funds received from all sources, period t, less repayments made during the same period.

ECBAL = The end of period cash balance, if positive.

BCBAL = The amount of cash on hand at the beginning of the analysis or at the end of the last period preceding the analysis. It is a parameter obtained from company records.

NMR_t= Various cash flows that are expected to occur, and for which an amount can be estimated, but which cannot be functionally related to a decision variable in the linear programming matrix.

c. Other Financial Constraints

It is difficult to identify other specific types of constraints that may be required in this section because they must be designed to satisfy the requirements of each individual model. Normally an inequality is required to set borrowing limits on each available external source of funds. Special or technical borrowing limitations may also require imaginative constraint formulation. This situation arose in regard to developing representative functions to incorporate the COMPA factoring arrangement described in detail in Section 4.3.1g.

d. Policy Constraints

Many firms adhere to financial and other operating policies that are stipulated by management or imposed by external authorities such as creditors, banks or government agencies. If these

policies tend to limit the options available to the firm they must also be formulated as constraints in the model. Typical policy constraints of this type would be required to represent borrowing limits, minimum liquidity ratios, labor relations objectives, inventory levels, etc.

3.7 Properties of the Functional

The objective function, cost equation or functional reflects management's objective in terms of available courses of action and the explicit and implicit cost or profit of each alternative. This objective may be either to maximize profit or minimize cost.

In the model solution or optimal plan the functional performs two significant services. First, it accumulates total cost or profit to be expected in a given situation. However, maximum profit or minimum cost as determined by the functional usually differs somewhat from figures found on operating statements or derived by accountants and it must be interpreted carefully. Second, and more important, the functional establishes the costs and benefits that control decision making. Where a decision variable has an explicit, quantifiable unit cost or unit revenue associated with its production, sale or storage, this measure is included in the functional for the model's use in the allocation of resources. The same considerations are valid for variables representing known external sources of funds or lending opportunities that can be assigned a specific interest cost coefficient. When explicit costs or interest rates do not apply but it is necessary to influence financial decisions in a particular way, these variables may be assigned implicit coefficients. The model can then use all financial costs and benefits provided to establish priorities that will insure needed funds are obtained at the least cost and excess cash allocated to the most profitable opportunity.

The maximizing form of the objective function:

$$\text{maximize } \pi = \sum_{t=1}^{M} \sum_{j=1}^{N} c_j Q_{jt} - \sum_{t=1}^{M} r\,(\text{EXTFUNDS}_t) + \sum_{t=1}^{M} r'\,(\text{ECBAL}_t) \qquad \text{(3-e)}$$

where:

Q_{jt} = See section 3.5a.

$EXTFUNDS_t$ = See section 3.5b.

$ECBAL_t$ = See section 3.5b.

c_j = Explicit cost or revenue derived from production, sale or storage of one unit of product j.

r = Explicit or implicit cost of external funds.

r' = Explicit or implicit rate at which excess cash may be loaned.

Chapter 4

COMPA: THE MODEL

4.1 Company Background

COMPA has been included in this study primarily because it has critical day-to-day cash management problems. The shortage of cash in this firm is quite severe and short-term financing is management's chief concern and the dominant consideration in every operating decision. Cash, rather than capacity, demand, etc., is the scarce resource that constrains COMPA's growth and profitability. For this reason, every dollar must be allocated to its most profitable use and an optimizing technique, e.g., linear programming, should be ideally suited to the task.

COMPA is a medium-sized manufacturer of confections. During its 30-year history it has actively competed in several segments of the candy industry. However, except for a few brief periods it has been only modestly profitable and occasional sharp losses have depleted working capital. As a result, routine payments have been deferred, trade credit stretched to the limit, and funds borrowed from any available source. The aftermath of these tactics for survival is a high-risk capital structure that excludes COMPA from conventional sources of financing. Although working capital is now obtained by factoring receivables, borrowing against inventory and using the bank float, these sources are not adequate to satisfy all needs. Unpaid taxes, overdue trade payables, and current requirements maintain an almost intolerable pressure on management to obtain additional funds or to improve utilization of available cash flow. Operating statements and balance sheets for 1964, 1965 and 1966 are included as Appendix A to this chapter to provide additional information concerning COMPA's operations and capital structure.

The COMPA product line includes over 100 individual items. However, the basic product is a coated confection made in nine sizes and product differentiation is based largely on packaging configurations. The production process is short and relatively simple. Each piece moves through five distinct steps during a five-day production cycle.

Two marketing divisions are responsible for sales. Division 1 sells to wholesale houses through a nationwide network of brokers. Division 2 sells directly to vending machine operators. In general neither the product lines nor the customers of the two divisions overlap. However, both are subject to a seasonal pattern of demand that reaches its peak in the summer and early fall and a low point in December and January. In addition, because candy wholesalers tend to pay promptly, and take the 2/10 cash discount that is offered, Division 1's receivables turn over faster than Division 2's.

Competition in the confectionery industry is intense. Although there are only a few companies that compete directly with COMPA, each must fight to keep its share of the market. For this reason, growth in sales is limited primarily to a stable share of the increased consumption that follows population expansion. Fads, freegoods, premiums and fast delivery are important competitive weapons in this industry and if more cash were available for promotion and to maintain larger stocks of finished goods, sales could probably be increased to some extent at competitors' expense.

COMPA operates two smaller manufacturing plants in addition to the main factory and home office. These plants are remote geographically and are organized as subsidiary corporations. They were not made an integral part of this study because both are largely autonomous in their operations and there is limited interaction between them and the parent. However, there are some minor and infrequent cash flows between COMPA and the subsidiaries that will be recognized in the model when they occur.

4.2 The Production Sub-Model

In planning this research it was hoped that a firm could be studied that was already using a linear programming model for production scheduling. Unfortunately, none could be found. Therefore, before the COMPA fully integrated model could be formulated it was necessary to develop a production sub-model.

4.2.1 Formulation of the Sub-Model

The first step in developing the sub-model was to identify those products that would become decision variables. For the COMPA model it was decided to include only products that had discernibly different physical characteristics or production costs. Products that were essentially identical were aggregated. This procedure reduced the array of products that had to be considered from over 100 to 46. Products, identified by codes, and net case weights of each are shown in Table 4.1.

The second major decision was to determine a common base in which to express production, sales and inventory of each product. It was found that for COMPA only net weight of finished product provided the necessary homogeneity. This is also a measure that is known for each product and the unit that management uses to gauge output.

As previously mentioned, the production process logically divides into five major steps. These are identified as MIX1, MIX2, FORM, COAT, and PACK1 or PACK2. MIX1 is the first step in the production process and the point at which basic ingredients are combined to form a base. Output from this process next goes to MIX2 where sugar is added and blended. The value added to work-in-process by MIX1 and MIX2 comprises about 25% of total direct labor cost and 65-75% of raw material cost. In FORM the base from MIX2 is shaped and molded into individual pieces. FORM is further sub-divided into two production lines, A and B. FORMA produces the smaller pieces and the larger ones are made by FORMB. Only labor cost is added during FORM.

Table 4.1
COMPA

Net Case Weight and Maximum Daily Production, by Product Code

Prod. code	Net Case weight, lbs	MIX1 (M1C)	MIX2[1] (M2C)	FORMA (S1C)	FORMB (S2C)	COAT (C1C)	PACK1 (B1C)	(B2C)	(B3C)	(B4C)	PACK2 #	(P#C)
Grp.'J'												
JB	20.00	3,520*	25,200*	n/a	14,400*	53,200*	n/a	n/a	13,300*	n/a		n/a
JC	38.46	"	"	"	"	"	"	"	"	"		"
JD	18.46	"	"	"	"	"	"	"	"	"		"
JF	36.15	"	"	"	"	"	"	"	"	"		"
JG	34.62	"	"	"	"	"	"	"	"	"		"
JI	34.36	"	"	"	"	"	"	"	"	"		"
JJ	33.33	"	"	"	"	"	"	"	"	"		"
JL	12.82	"	"	"	"	"	"	"	"	"		"
JN	12.82	"	"	"	"	"	"	"	"	"		"
JO	14.74	"	"	"	"	"	"	"	"	"		"
JP	15.38	"	"	"	"	"	"	"	"	"		"
JR	25.77	"	"	"	"	"	"	"	"	"		"
JX	27.69	"	"	"	"	"	"	"	"	"	3	776*
JY	37.00	"	"	"	"	"	"	"	"	"	3	829
Grp.'S'												
SB	25.00	3,520	25,200	n/a	14,400	49,300	n/a	n/a	n/a	n/a		n/a
SD	24.62	"	"	"	"	"	6,400*	"	9,600	"		"
SE	24.62	"	"	"	"	"	n/a	6,400*	n/a	"		"
SF	44.31	"	"	"	"	"	6,400	n/a	9,600	"		"
SH(s)	9.85	"	"	"	"	"	"	"	"	"		"
SI(s)	9.85	"	"	"	"	"	n/a	6,400	n/a	"		"
SJ(s)	14.77	"	"	"	"	"	6,400	n/a	9,600	"		"
SL	37.00	"	"	"	"	"	3,432	"	"	"		"
SM	18.50	"	"	"	"	"	"	"	"	"		"
SN	7.38	"	"	"	"	"	n/a	n/a	n/a	n/a	2	960*
Grp.'K'												
KB(2)	18.00	3,520	12,600	"	12,600	50,400	n/a	n/a	n/a	18,000*		n/a
KC	32.70	"	"	"	"	"	"	"	"	n/a	7	294*
KD	8.94	"	"	"	"	"	"	"	"	18,000		n/a
KE	17.87	"	"	"	"	"	"	"	"	"		"
KF	6.55	"	"	"	"	"	1,120	"	"	n/a		"
Grp.'T'												
TA	31.20	n/a	n/a	n/a	n/a	54,200	n/a	n/a	n/a	n/a	6	2,056*
TC	34.30	"	"	"	"	"	3,200	"	"	"	6	"
TD	17.20	"	"	"	"	"	"	"	"	"	6	"
Grp.'B'												
BA(2)	25.00	3,520	14,400	14,400	n/a	34,300	n/a	n/a	n/a	24,500		n/a
BB(2)	25.00	"	"	"	"	"	"	"	"	"		"
BC(2)	25.00	"	"	"	"	48,400	"	"	"	22,000		"
BD(2)	23.00	"	10,800	10,800*	"	41,000	"	"	"	"		"
BF	15.40	"	"	"	"	49,000	"	"	"	n/a	4	1,696
BG	28.80	"	"	"	"	"	"	"	"	"	4	1,088*
BI	32.00	"	"	"	"	"	3,200	"	"	"	4	1,725
BJ	16.00	"	"	"	"	"	"	"	"	"	4	"
Grp.'M'												
MA(2)	40.00	3,520	14,400	n/a	n/a	n/a	n/a	n/a	n/a	n/a	1	1,200*
MB(2)	25.00	"	"	14,400	"	56,000	"	"	"	"	5	2,000*
MC	18.46	"	5,400	n/a	5,400	50,400	3,376	"	"	"		n/a
MD	34.30	"	25,200	"	14,400	42,000	5,920	"	"	"		"
ME(2)	25.00	n/a	n/a	"	n/a	62,440	n/a	"	"	"		"
MF(2)	25.00	3,520	25,200	"	14,400	31,920	"	"	"	"		"

* = B_i* [1] = This process is not included in the production sub-model, Appendix B.

(s) = Special seasonal product (2) = Products marketed by division 2

After one or two days of curing, the pieces are finished by covering with a candy coating in the COAT process. At this point all pre-packaging production costs have been incurred and the finished product is ready to be packed. All bulk and standard items are wrapped and packed on one of the four lines in PACK1. Specialty and low volume items are packed in PACK2 on one of seven special purpose lines.

In order to formulate a production model, constraint inequalities similar to (3-a) were developed to represent each step in the process. However, before discussing how the values for each coefficient and right hand side were derived, it will be convenient to consider unit raw material and labor costs. Fully allocated costs are seldom used in linear programming analyses. Arbitrary distribution of overhead distorts relative contribution or the true ordering of marginal profitability among products and only direct costs are used in the COMPA model. Direct costs have been accumulated in three categories: labor, sugar, and other raw materials, including packaging supplies and premiums.

A complete explanation of how each cost element and capacity utilization factor was calculated would be extremely time-consuming and largely repetitious. In addition, this information would not have had to be developed if it could have been provided by COMPA from existing records. Therefore, the complete results of this part of the study are shown only as final coefficients in the initial tableau of the sub-model (Appendix B). To illustrate how coefficients were determined for product group "K", the derivation of resource consumption factors is shown in Table 4.2 and calculation of direct cost elements in Table 4.3.

When process capacities were calculated it was apparent that there is considerable excess in all processes if capacity is defined in terms of the maximum output that could be expected from complete utilization of existing production facilities. Also, output from each process can be increased from current levels by adding manhours to existing shifts without the need for more capital equipment. For this reason it seemed more appropriate to estimate maximum normal daily capacity in each process as the volume of output possible from an eight to 16-hour workday during which the optimal size

Table 4.2
COMPA

Calculation of Resource Consumption Coefficients
Product Group "K"

	MIX1	*MIX2*	*FORMB*	*COAT*	*PACK1 (KB)*	*PACK2 (KC)*
Normal hours per day	10	16	16	16	8	8
Capacity per day, lbs.	3,520	12,600	12,600	50,400	18,000	294
Manhours per day	20	42.5	67	333	87	43
Pounds per manhour	176	296	188	151	206	6.85
Ratio: End of process weight to finished weight	.115	.722	.722	1.	1.	1.
Finished lbs. per manhour	1,540	410	260	151	206	6.85
Manhour per pound	.00066	.00244	.00385	.00661	.00485	.1457

workforce, consistent with balanced production, would be employed. The upper limits that were determined by this analysis are shown in Table 4.1. However, even at the maximum normal level, there was such a large amount of excess capacity indicated for MIX2 that it was eliminated from the sub-model as redundant.

Labor consumption coefficients were determined by analyzing each process to determine the average output per manhour at normal volumes. No attempt was made to set rates assuming maximum efficiency. Rather, the objective was to reflect productivity that could be expected from the existing workforce. An illustration of how labor consumption factors were calculated is provided in Table 4.2.

Unit costs were accumulated for each product in the three categories: labor, sugar, and other raw material. Sugar cost is determined separately because it comprises over one-half of total raw material cost for most products and because sugar payments have distinct cash flow characteristics that will be reflected in the fully integrated model. Each product was traced through applicable processes and the value added in each was determined. Value added per pound of work-in-process at each step was

Table 4.3
COMPA

Calculation of Direct Unit Costs
Product Group "K"

	Ratio	*Labor*	*ORM*	*Sugar*
Value added, per pound:				
MIX1	n.a.	$.0123	$.167	$ -
MIX2	n.a.	.00743	.0164	.0775
FORM	n.a.	.01065	-	-
COAT	n.a.	.016	.0048	.0332
Converted to finished wt.				
MIX1	.115	.00141	.0192	-
MIX2	.722	.00535	.0118	.0560
FORM	.722	.0077	-	-
COAT	1.0	.016	.0048	.0332
Sub-total:		.03046	.0358	.0892

Total direct production cost per pound of finished product:	Direct labor	$.03046
	Other raw material	.03580
	Sugar	.08920
	Total:	$.15546

Calculation of individual product cost, Product Group "K"

	KB	*KC*	*KD*	*KE*	*KF*
Candy piece	.15546	.15546	.15546	.15546	.15546
Packing Cost:					
Direct labor	.00666	.16700	.01225	.01255	.10800
Material	.01040	.13380	.02267	.03440	.15000
Premiums	-	-	-	.35000	.12800
Total direct cost:	.17252	.45526	.19068	.55241	.54146
Total direct cost by cost category:					
Direct labor (DLC)	.03712	.19746	.04301	.04301	.13846
Other raw material (ORM)	.04620	.16960	.05847	.42020	.31380
Sugar (SUG)	.08920	.08920	.08920	.08920	.08920
Total direct cost:	.17252	.45626	.19068	.55241	.54146

converted to cost per pound of finished product. This revision is illustrated in Table 4.3 in the unit cost calculations for product group "K".

Before the production sub-model is tested, a few special aspects of its formulation and assembly should be mentioned. First, in Table 4.1, the maximum capacity of each process is expressed in terms of every product. That is, if the total capacity of process "i" was devoted exclusively to the production of product "j", B_{ij} pounds could be completed during one working day. However, these are work-in-process pounds and the common base that was selected is net weight of finished product. In addition, only one upper limit can be specified for each process. For this reason one B_{ij} had to be arbitrarily selected as the process upper limit. This value was designated B_i^*. In process "i" the coefficient a_{ij}^* for the base product j* is equal to the ratio of product j*'s weight at the completion of the process (w_i) to its finished weight (w_f). For example, if $(w_i/w_f) = q_j^*$; $q_j^* = a_{ij}^*$. For products other than j* that pass through process "i", the ratio $q_j \neq a_{ij}$. This result follows because products are processed at different rates. To compensate for this fact, q_j must be modified by the ratio (B_i^*/B_{ij}). That is, $a_{ij} = q_j(B_i^*/B_{ij})$. An example of this adjustment can be found by reference to Appendix B and Tables 4.1 and 4.2. In Table 4.2, process S2C, the ratio $q_j = .722$. However, in Appendix B, the coefficient a_{ij} for process S2C, products KB to KF = .825. In Table 4.1, B_i^* for process S2C - 14,400 pounds per day, j* = products in group "J". B_{ij} for product group "K" is 12,600 pounds in this process and a_{ij} is found to be:

$$a_{ij} = q_j(B_i^*/B_{ij}) = .722\,(14400/12600) = .825$$

Referring again to Appendix B, it should be noted that the labor consumption constraints for MIX1 and MIX2 have been combined to form M1L and those for PACK1 and PACK2 have been combined to form B1L. Although the analysis that led to these aggregations will not be explained in detail, it was found that there was considerable informal transfer of workers within each pair of processes. Because of this, the labor hour records that were used to provide bench marks for testing the sub-model did not accurately reflect the manhours used in the individual processes.

4.2.2 Validating the Sub-Model

Before the COMPA production sub-model (Appendix B) could be used with confidence as the base for a fully integrated model, its representativeness had to be validated. Testing was also required to determine if actual production/consumption relationships are approximately linear. To provide the necessary points of comparison for these tests, actual consumption of the various factors was determined at several levels of production. Also, to insure that the model was validated over the largest possible range, it was necessary to include periods of maximum and minimum production. For this reason, the 1966 months of maximum output (August), minimum output (February), and the three most recent months for which data were available (September, October, and November), were selected as test periods.

Testing was based on the assumption that if the model was forced to simulate production of the exact amount and mix of items produced during the base periods, predicted levels of factor consumption and cost should be approximately the same as actual. Because these tests were simulations, optimization was not required and a specific functional was not formulated for the sub-model. For this reason solution values were usually not obtained by using linear programming algorithms. Rather, a computer program was written to multiply each row of the matrix by vectors that contained the quantity of every item produced in the base periods. The inner products of these multiplications provided most of the model values used in the tests.

The most difficult actual values to determine were the manhours worked in each process. This information had to be accumulated from weekly time records and allocated to process output after insuring that inherent in-process lags were explicitly recognized. For example, February output included labor value added in MIX1 during the last week in January and the first three weeks in February. Material processed through MIX1 the last week in February would ultimately be included in March transfers to finished goods. In contrast, because packing is the last production step, PACK1 and PACK2 hours worked in a period can be assumed to be applied to output for the same period.

Available time records included straight time, overtime, direct and indirect hours worked. To obtain an adequate measure of hours directly applicable to production it was necessary to eliminate all time devoted to indirect or overhead functions. It was also necessary to separate hourly (variable) and supervisory (fixed) hours. Although COMPA's foremen were working supervisors and most of their time was properly charged to production, they were also guaranteed a minimum work week that kept direct labor from being completely variable.

The first series of tests was designed to establish that the relationships between output and hours worked, sugar usage and other raw material usage is approximately linear and that the model can provide representative estimates of consumption of these factors. Direct labor cost estimates provided by the model were not tested because actual payroll expenditures could not be disaggregated accurately enough to provide check values. Model estimates compared with actual values for each of the five test periods are shown in Table 4.4. These comparisons indicate that in most instances the model value closely approximates actual and in only a few cases are differences as large as 10%.

To establish that actual production functions are approximately linear in the relevant range and also to help give perspective to the actual/model comparisons, each group of observations was plotted as shown in Figure 4.1. In addition, the observations have been used in a series of simple linear regression analyses. The resulting regression equations and lines of least squares are also shown in Figure 4.1. Because the regression equations are based on only five observations, no attempt was made to establish levels of significance for the regression coefficients. However, coefficients of determination (r^2) were calculated for each equation. The values of r^2 are above .8 in every instance and several are above .9. The causal relationships indicated by the high r^2 values and the approximate linearity indicated by the plots of actual values establish that the COMPA production process can be represented by a linear model.

These tests also denote that the production sub-model is reasonably representative of the production process. Reference to Figure 4.1 shows that point estimates closely approximate actual specific

Table 4.4
COMPA

Comparison of Factor Consumption Predicted by the Production Sub-Model with Actual Factor Consumption

	February			August			September			October			November		
	Actual	Model Est.	% Diff.	Actual	Model Est.	% Diff.	Actual	Model Est.	% Diff.	Actual	Model Est.	%Diff.	Actual	Model Est.	% Diff.
Direct labor: (m/h)															
MIX1 ≠ MIX2 (M1L)	1,267	1,133	(10.5)	1,977	1,934	(2.0)	1,691	1,734	2.5	1,531	1,662	8.5	1,547	1,466	(5.2)
FORM (SOL)	1,484	1,370	(7.6)	2,389	2,357	(1.4)	2,021	2,141	6.0	2,045	2,016	(1.0)	2,012	1,842	(8.2)
COAT (C1L)	2,818	2,721	(3.2)	4,613	4,787	3.8	4,000	4,249	6.2	4,162	4,184	.5	4,044	3,646	(9.7)
PACK1 ≠ PACK2 (B1L)	3,447	3,136	(9.0)	5,795	5,716	(1.3)	5,537	5,577	.8	4,575	4,943	8.0	4,330	4,217	(2.8)
[1]Other raw material: ($)															
Debit to inv.	33,430	26,630	(20.2)	43,197	45,654	5.8	39,313	43,136	10.0	48,716	40,467	(17.0)	40,268	38,181	(5.2)
Credit to inv.	28,599	26,630	(6.8)	42,296	45,654	8.0	44,817	43,136	(3.7)	37,700	40,467	7.2	34,238	38,181	11.2
[1]Sugar: ($)															
Debit to inv.	31,229	32,063	2.7	57,363	55,328	(3.5)	50,116	49,842	(.6)	49,178	48,210	(2.0)	38,371	42,768	11.2
Credit to inv.	33,338	32,063	(3.8)	51,636	55,328	7.0	52,558	49,842	(5.2)	48,004	48,210	.4	40,607	42,768	5.1
[2]Total weight:	388,625	388,561	-	674,408	674,408	-	614,744	614,744	-	586,319	586,319	-	519,699	519,684	-

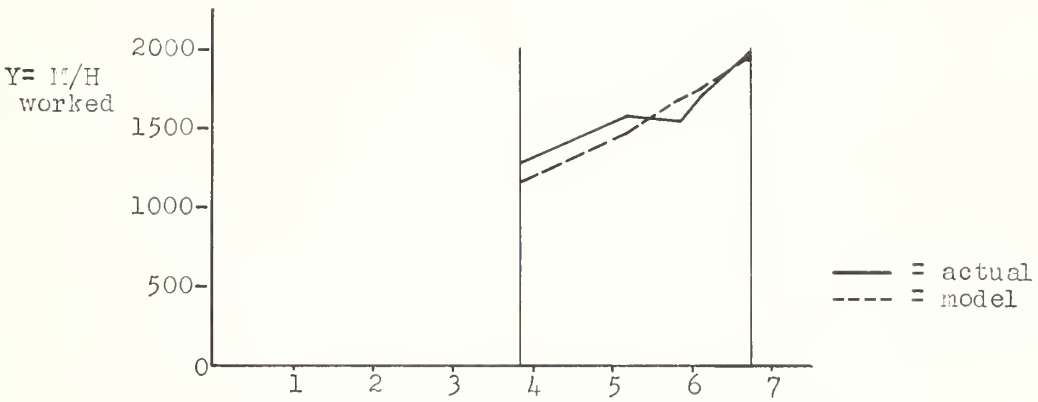
1, Debits to inventory are purchases recorded during the period. Credits to inventory represent the cost of material transferred to work-in-process. Because the model predicts the cost of materials used in production, it should provide estimates most closely related to credits to inventory.

2, Minor differences between actual and estimated weights are due to rounding.

Figure 4.1 (a)
COMPA

Actual/Model Comparisons:
Manhours Worked

Process: MILL



The relationship between actual manhours worked and manhour requirements predicted by the production sub-model.

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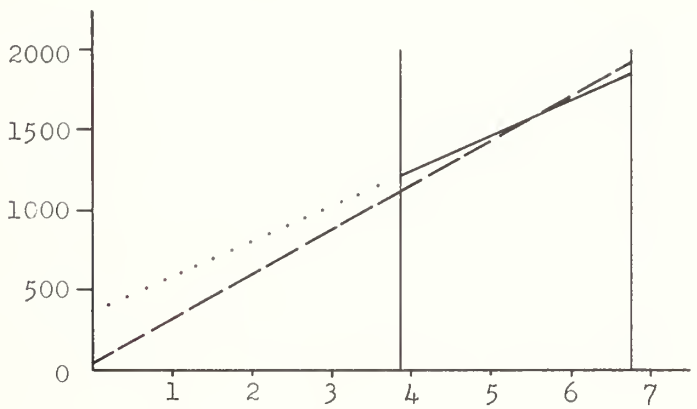


Regressions of actual manhours worked and manhours predicted by the model on total pounds produced.

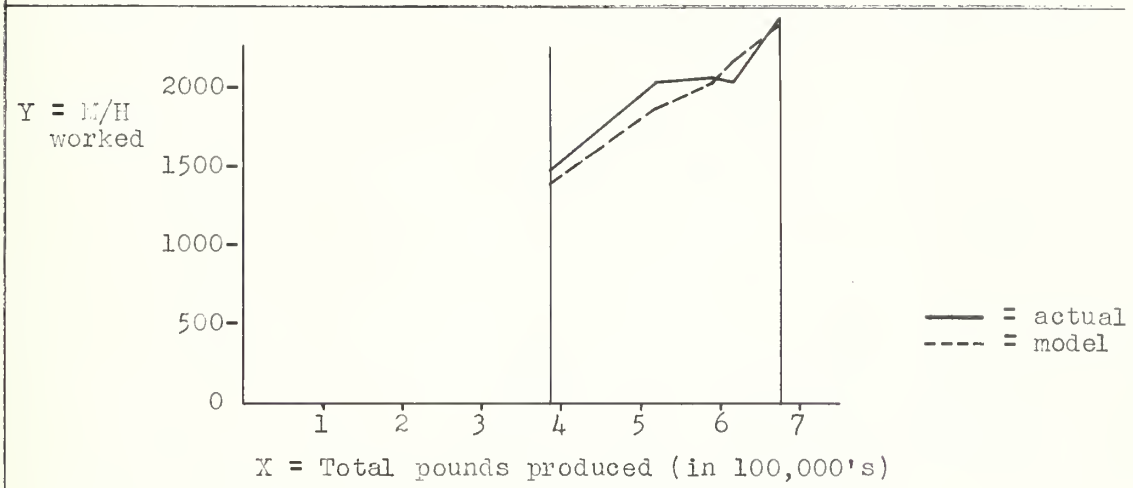
Equations: actual, $Y = 371.198 + .00221173X$ $r^2 = .868316$

model, $Y = 40.0927 + .00277662X$ $r^2 = .996779$

Figure 4.1 (b)
COMPA

Actual/Model Comparisons:
Manhours Worked

Process: SOL



The relationship between manhours worked and manhour requirements predicted by the production sub-model.

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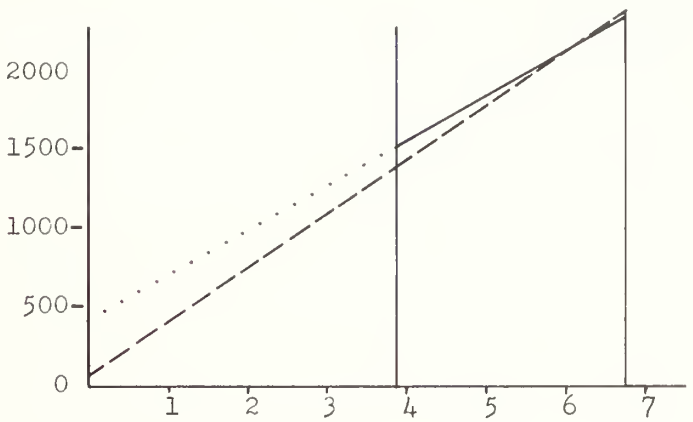


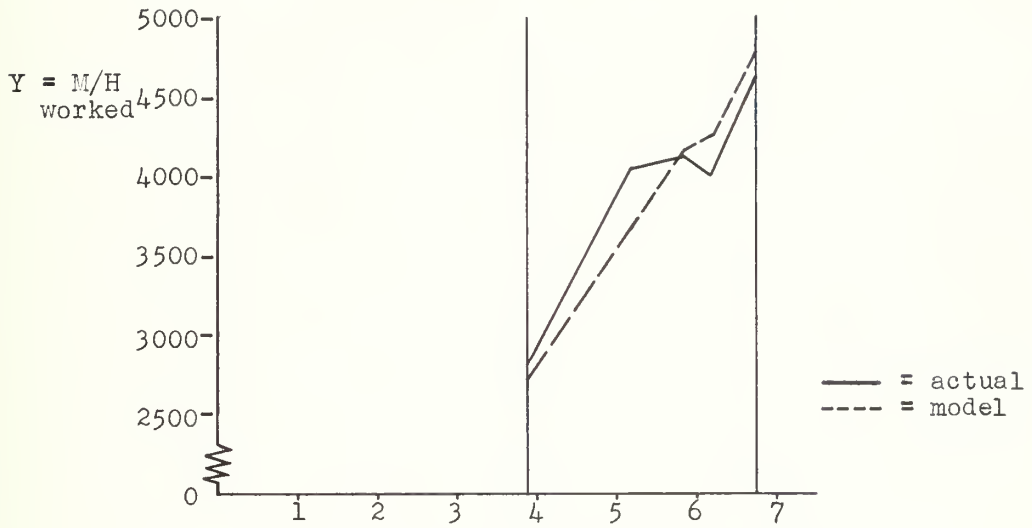
Regressions of actual manhours worked and manhours predicted by the model on total pounds produced.

Equations: actual, $Y = 419.488 + .00282117X$ $r^2 = .904923$
model, $Y = 50.5642 + .0034035X$ $r^2 = .997197$

Figure 4.1 (c)
COMPA

Actual/Model Comparisons:
Manhours Worked

Process: C1L



X = Total pounds produced (in 100,000's)

The relationship between actual manhours worked and manhour requirements predicted by the production sub-model.

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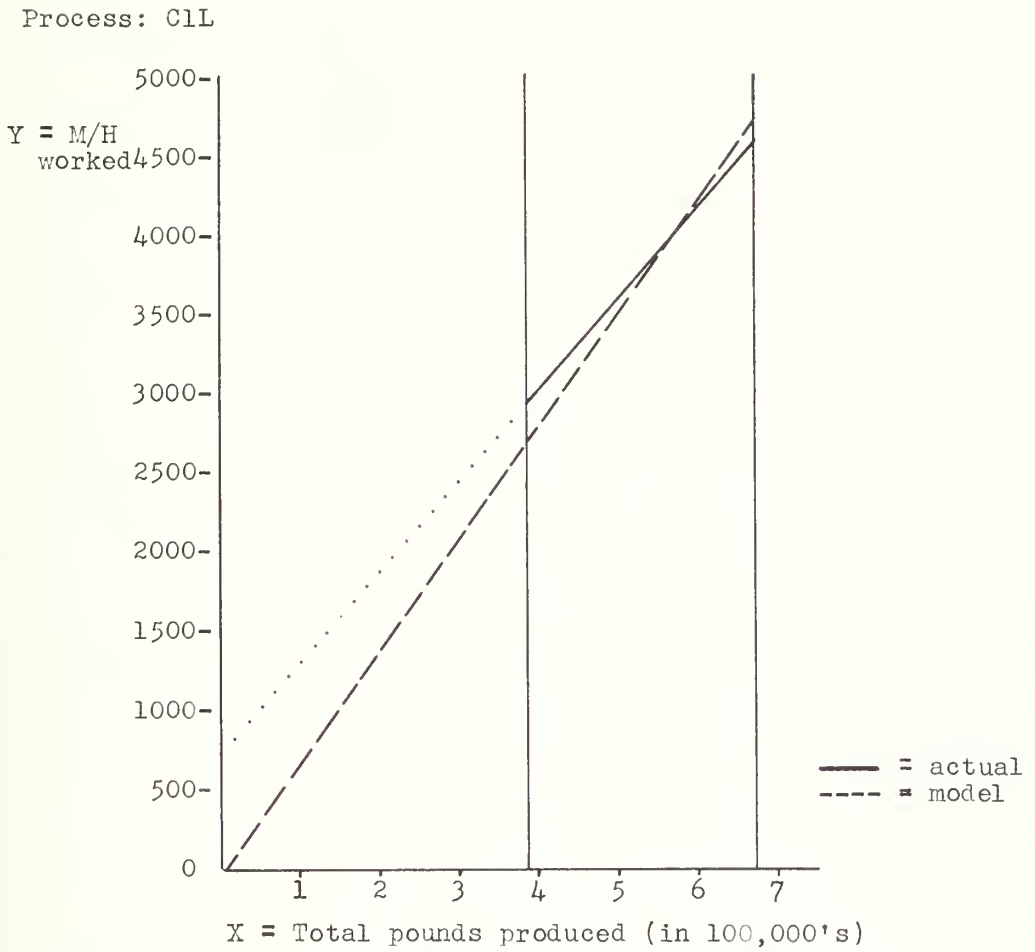
(Regression equations for process C1L are on the following page)

Figure 4.1 (c) (continued)



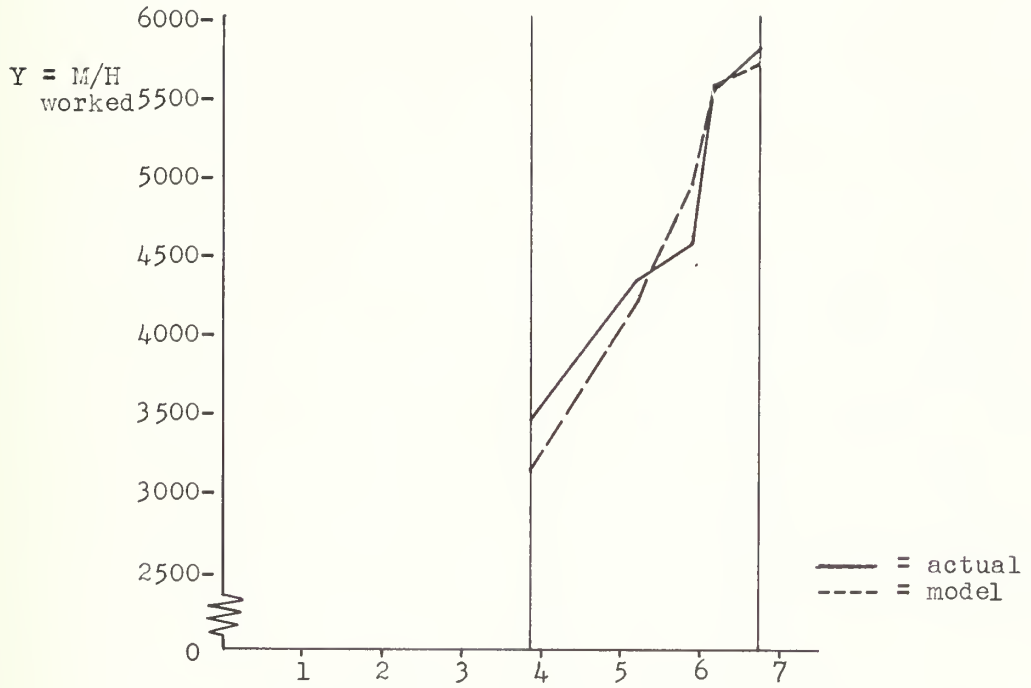
Regressions of actual manhours worked and manhours predicted by the model on total pounds produced.

Equations: actual, $Y = 730.442 + .00574209X$ $r^2 = .886561$
model, $Y = -55.2779 + .00713521X$ $r^2 = .995502$

Figure 4.1 (d)
COMPA

Actual/Model Comparisons:
Manhours Worked

Process: B1L



The relationship between actual manhours worked and manhour requirements predicted by the production sub-model.

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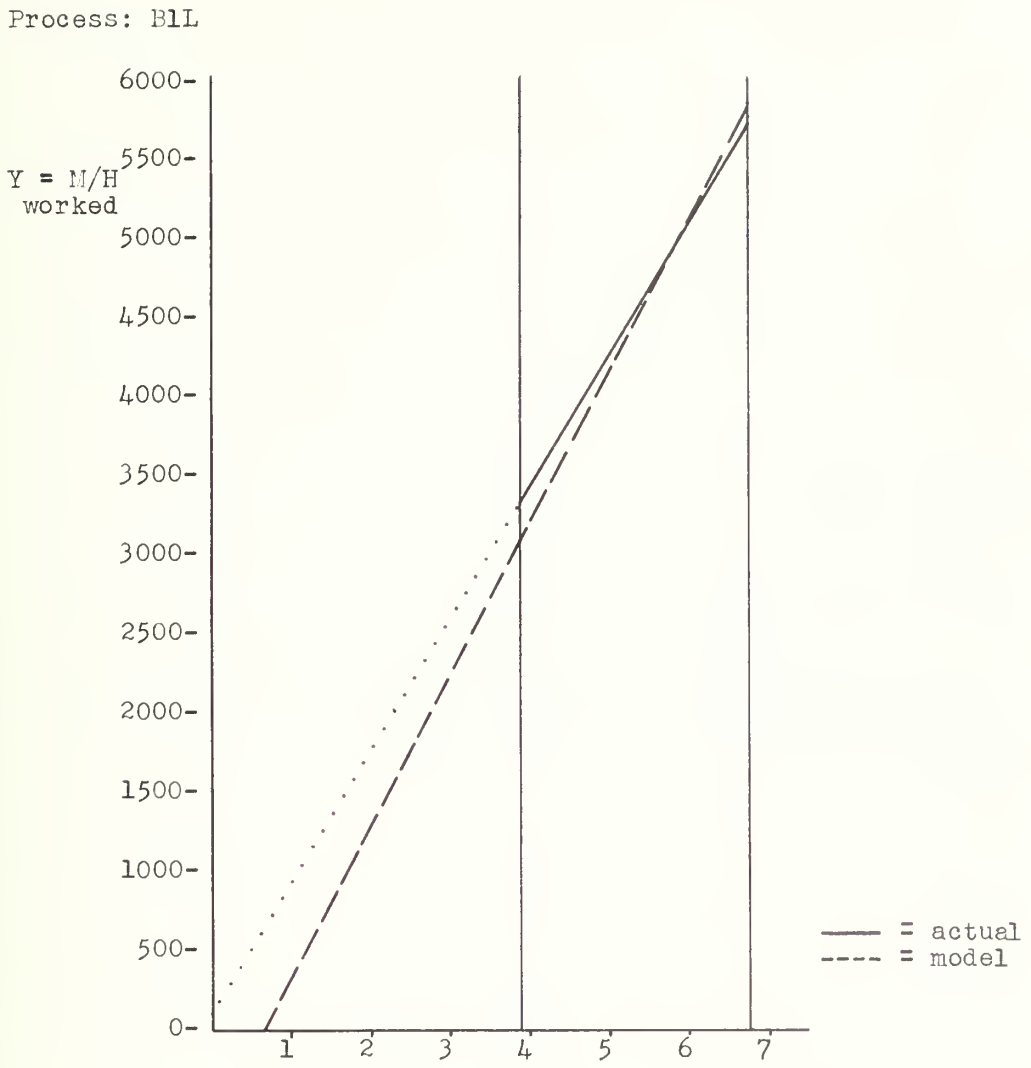
(Regression equations for process B1L are on the following page)

Figure 4.1 (d) (continued)



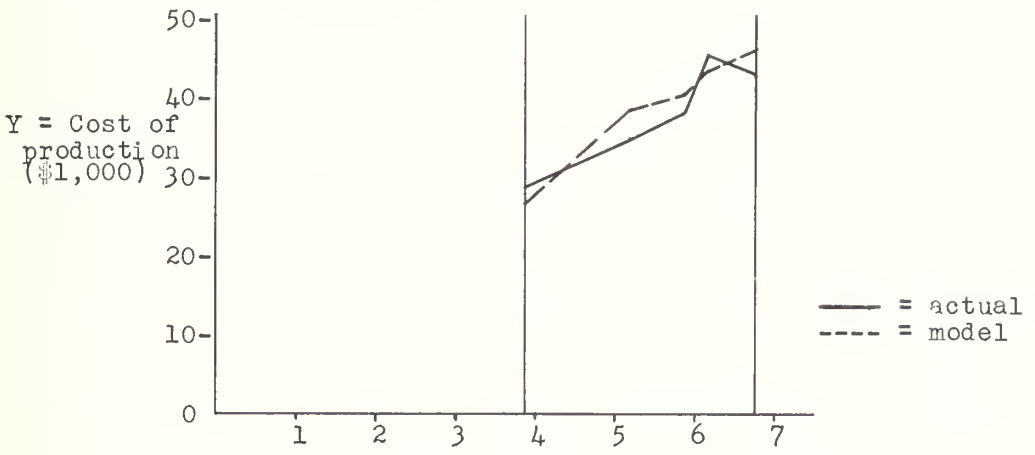
Regressions of actual manhours worked and manhours predicted by the model on total pounds produced.

Equations: actual, $Y = 92.6732 + .00834136X$ $r^2 = .918378$
model, $Y = -630.016 + .0096015X$ $r^2 = .970359$

Figure 4.1 (e)
COMPA

Actual/Model Comparisons:
Direct Costs

Cost item: Other Raw Material (ORM)



The relationship between actual cost of other raw materials consumed and cost of other raw material predicted by the model.

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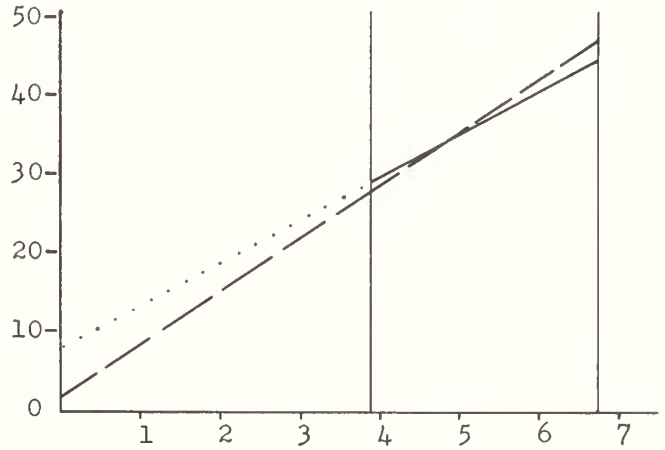


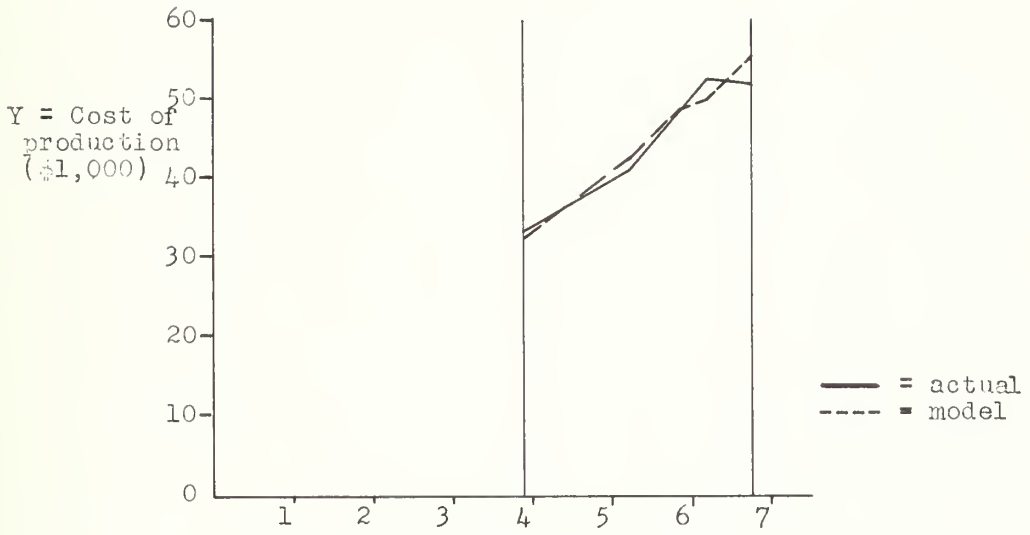
Regressions of actual cost of other raw materials consumed and cost predicted by the model on total pounds produced.

Equations: actual, $Y = 7056.51 + .0547337X$ $r^2 = .858896$
model, $Y = 1753.97 + .0665628X$ $r^2 = .973995$

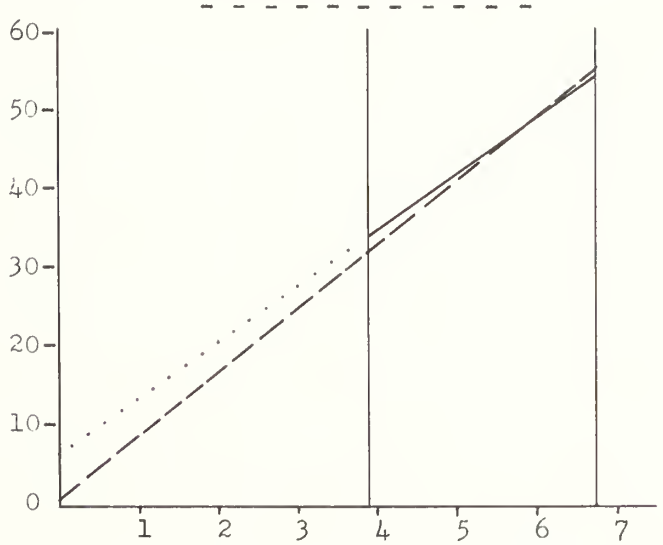
Figure 4.1 (f)
COMPA

Actual/Model Comparisons:
Direct Costs

Cost item: Sugar (SUG)



The relationship between actual cost of sugar consumed and cost of sugar predicted by the production sub-model.



Regressions of actual cost of sugar consumed and cost predicted by the model on total pounds produced.

Equations: actual, $Y = 6560.77 + .0702279X$ $r^2 = .945248$
model, $Y = 740.14 + .0806486X$ $r^2 = .998988$

levels of output, that regression coefficients in the model equations are close to those in the actual equations, and that the lines of least squares cross within the range of observations. This crossing was expected because the homogeneity present in the model causes the regression lines based on model estimates to intercept the vertical axis closer to the origin.

The final series of tests was designed to determine if process capacity constraints should be included in the fully integrated model. Because actual proportions of process capacity employed could not be determined from company records, comparisons were made between the calculated upper limits for each process (B_1* from Table 1) and the amount of capacity used by the model in simulating each of the five test periods. These comparisons are shown in Table 4.5 and indicate that in only a few instances was as much as 90% of estimated capacity used in any period. In fact, during August, the 1966 month of peak production, utilization of only two processes exceeded 90% of capacity. For this reason and because upper limits are estimates of normal, rather than true maximum capacity, it appears unlikely that capacity will be a binding constraint within the short-term horizon of this analysis. Therefore, capacity constraints are redundant and will not be included in the fully integrated model.

4.3 The Fully Integrated Model

Development of the COMPA fully integrated, multiperiod, linear programming model will be discussed in this section. As initially formulated the model will represent all important production, sales, and financial decisions required during a four-period (month) planning cycle. Although any number of periods, of equal of unequal length, could be included in the model, it will be more convenient to test if its size is limited. However, a four-period planning cycle is adequate to permit testing of the model's dynamic features.

Table 4.5
COMPA

Comparison of Capacity Utilization Predicted by the Production Sub-Model with Estimated Maximum Normal Capacity (pounds)

	February (19 days)			August (23 days)			September (21 days)			October (20 days)			November (20 days)		
	Maximum	Model est.	% used	Maximum	Model est.	% used	Maximum	Model est.	% used	Maximum	Model est.	% used	Maximum	Model est.	% used
Process:															
MIX1 (M1C)	66,880	43,857	65.5	80,960	74,550	92.0	73,920	67,812	91.8	70,400	63,932	90.8	70,400	57,581	81.8
FORMA (S1C)	205,200	109,319	53.3	248,400	181,646	73.0	226,800	139,045	61.4	216,000	156,829	72.5	216,000	123,137	57.0
FORMB (S2C)	273,600	154,062	56.4	331,200	278,742	84.1	302,400	279,523	92.5	288,000	236,949	82.3	288,000	232,250	80.8
COAT (C1C)	1,010,800	424,795	42.0	1,223,600	749,881	61.2	1,117,200	667,490	60.0	1,064,000	648,843	60.9	1,064,000	575,640	54.0
PACK1 (B1C)	121,600	34,164	28.1	147,200	87,719	59.5	134,400	105,781	78.5	128,000	70,115	54.8	128,000	82,196	64.2
(B2C)	121,600	10,039	8.3	147,200	21,936	14.9	134,400	13,393	10.0	128,000	22,269	17.4	128,000	38,651	30.2
(B3C)	252,700	110,611	43.8	305,900	162,831	53.1	279,300	182,155	65.3	266,000	145,141	54.5	266,000	169,500	63.7
(B4C)	342,000	210,079	61.5	414,000	395,807	96.1	378,000	345,084	91.2	360,000	341,020	94.8	360,000	273,134	76.0
PACK2 (P1C)	22,800	6,230	27.4	27,600	2,720	9.8	25,200	2,355	9.3	24,000	2,000	8.3	24,000	840	3.5
(P2C)	18,240	1,538	8.5	22,080	0	0	20,160	3,410	16.9	19,200	1,616	8.4	19,200	0	0
(P3C)	14,744	111	.8	17,848	3,930	22.0	16,296	1,190	7.3	15,520	1,052	6.8	15,520	1,412	9.1
(P4C)	20,672	7,345	35.6	25,024	22,413	89.5	22,848	20,771	91.0	21,760	19,703	90.5	21,760	12,838	59.0
(P5C)	38,000	16,850	44.3	46,000	1,450	3.1	42,000	5,850	14.0	40,000	8,700	21.8	40,000	6,875	17.2
(P6C)	39,064	953	2.5	47,288	10,153	21.4	43,176	11,253	26.2	41,120	4,430	10.8	41,120	2,402	5.8
(P7C)	5,586	0	0	6,762	1,046	95.5	6,174	948	15.3	5,880	0	0	5,880	98	1.7

The general notation that will be used for variables in the model is defined as follows:

P_{jT} = Total production of product j, in period T, expressed in pounds of net finished weight.

S_{jT} = Total sales of product j, in period T, expressed in pounds of net finished weight.

B_{jT} = Total inventory of product j, at the end of period T, expressed in pounds of net finished weight.

RT_{iT} = Number of hours worked, at straight time rates, in process i during period T.

ET_{iT} = Number of hours worked, at overtime rates, in process i during period T.

UT_{iT} = Number of hours idle, but paid at straight time rates, in process i during period T.

GUS_T = Direct cost of sugar consumed in production during period T.

RRM_T = Direct cost of all raw materials, other than sugar, used during period T.

WGJ_T = Total weight shipped by Division 1 during period T.

WGC_T = Total weight shipped by Division 2 during period T.

$SJ\$_T$ = Gross sales revenue of Division 1 during period T.

$SC\$_T$ = Gross sales revenue of Division 2 during period T.

$FP\$_T$ = Total debits to the factor's account during period T.

$FB\$_T$ = Credit balance in the factor's account at the end of period T.

$CA\$_T$ = Cash balance at the end of period T.

$LB\$_T$ = Implicit overdraft at the end of period t.

All variables are constrained to non-negativity.

4.3.1 Constraint Equations and Objective Functions

a. Production, Sales and Inventory Balance:

In the COMPA model these constraints have the following form:

$$B_{j0} = \sum_{t=1}^{T} P_{jt} - \sum_{t=1}^{T} S_{jt} = B_{jT}$$

or (1)

$$-\sum_{t=1}^{T} P_{jt} + \sum_{t=1}^{T} S_t + B_{jT} = B_{j0}$$

$T = 1,M \qquad j = 1,N$

b. Direct Labor Consumption:

In the COMPA model the constraints that accumulate and limit direct labor hours in each process have the form:

$$\sum_{j=1}^{N} l_{ij} P_{jT} = RT_{iT} + ET_{iT} + FT_{iT}$$

or

$$\sum_{j=1}^{N} l_{ij} P_{jT} - RT_{iT} - ET_{iT} = FT_{iT} \qquad (2)$$

$$i = M1L, SOL, C1L, \text{ or } B1L \qquad T = 1,M$$

where:

l_{ij} = The amount of direct labor hours used in process i to produce one pound of product j. Values for these coefficients were determined for the production sub-model and are found in Appendix B. The subscript i identifies the process, M1L, SOL, C1L, or B1L.

FT_{iT} = The total supervisory (fixed) manhours that were applied directly to production in process i during period T. This quantity is the product of an average daily rate for each process multiplied by the number of working days in the period. For COMPA, average daily direct supervisory manhours are estimated to be:

process: M1L = 24
SOL = 18
C1L = 27
B1L = 22

These equations have three functions in the model. First, the initial term calculates total manhours required in each process to manufacture the desired quantity and mix of products. Second, they provide each process with a quantity of straight time manhours and require that overtime be used after straight time is exhausted. Third, supervisory hours, that are direct but also fixed, are deducted from total hours worked so that only variable hours are distributed between straight time and overtime.

c. Direct Cost of Sugar:

This constraint accumulates the total direct cost of sugar used in production during each period. It has the form:

$$\sum_{j=1}^{N} s_j P_{jT} = GUS_T \qquad (3)$$

or

$$\sum_{j=1}^{N} s_j P_{jT} - GUS_T = 0 \qquad (3)$$

$$T = 1,M$$

where:

s_j = The total cost of sugar required to produce one pound of product j. Values for s_j are shown in Appendix B.

d. Direct Cost of Other Raw Material:

This constraint accumulates total direct cost of all raw materials, other than sugar, used during each period. It is identical in form to equation (3).

$$\sum_{j=1}^{N} r_j P_{jT} = RRM_T$$

or (4)

$$\sum_{j=1}^{N} r_j P_{jT} - RRM_T = 0$$

$$T = 1,M$$

where:

r_j = The total cost of all raw materials, other than sugar, required to product and pack one pound of product j. Values for r_j are shown in Appendix B.

e. Net Weight of Shipments:

These constraints are used to accumulate total weight shipped during each period. However, because the average cost of freight out is slightly higher for Division 1 than for Division 2, it is necessary to accumulate weight shipped by division. The constraint equations appear as follows:

Division 1: $\sum_{j=1}^{N-L} S_{jT} = WGJ_T$

or (5A)

$$\sum_{j=1}^{N-L} S_{jT} - WGJ_T = 0$$

$T = 1,M$ $\qquad$ $N - L$ = Number of products sold by Division 1.

Division 2: $\sum_{j=N-L+1}^{N} S_{jT} = WGC_T$

or (5B)

$$\sum_{j=N-L+1}^{N} S_{jT} - WGC_T = 0$$

T = 1,M L = Number of products sold by Division 2.

It should be noted that the variables S_{jT} are net weight although freight charges are usually a function of gross weight shipped. However, coefficients have not been included in (5A) and (5B) to convert net to gross weight. This adjustment is included in the freight rate coefficients used in the cash flow constraint and the functional.

f. Total Sales Revenue:

These constraints accumulate total sales revenue by division for each period. They have the form:

Division 1: $\sum_{j=1}^{N-L} sp_j S_{jT} = SJ\$_T$

or (6A)

$$\sum_{j=1}^{N-L} sp_j S_{jT} - SJ\$_T = 0$$

T = 1,M N – L = Number of products sold by Division 1.

Division 2:

$$\sum_{j=N-L+1}^{N} sp_j S_{jT} = SC\$_T$$

or (6B)

$$\sum_{j=N-L+1}^{N} sp_j S_{jT} - SC\$_T = 0$$

T = 1,M L = Number of products sold by Division 2.

where:

sp_j = The average gross selling price per pound of product j. For products sold by Division 1, sp_j is list price per pound. For Division 2's products it is an average figure somewhat less than list price. This difference reflects Division 2's practice of using "free goods" for promotional purposes

Because COMPA is limited primarily by funds, none of the six sets of constraints that have been formulated are directly binding. However, equation (2) can be used as an upper limit by specifying

maximum straight time and/or overtime manhours. The next equations that will be discussed form the financial section of the model. Because the fully integrated model will be tested by a simulation procedure, similar to the one used to validate the sub-model, it will initially include equations representing the actual sources of external financing used during the July-October 1966 test period, i.e., factoring and overdrafts. After testing has been completed, the model can be modified to reflect various financing alternatives that management may desire to evaluate.

g. Factoring:

The factoring agreement authorizes COMPA to receive cash immediately for 80% of net amount billed for all receivables sold to the factor. The company agrees to repurchase all receivables that are unpaid after 90 days. In practice, all receivables are factored and all payments received from customers are transmitted to the factor. Daily, the factor offsets debits and credits, determines the net amount due, and transfers it to COMPA's bank. An example of these transactions, using "T" accounts to represent COMPA's records, appears as follows:

Sales		Accounts Rec.		Cash		Factor	
	100(1)	100(1)	50(2)	80(1)		50(2)	80(1)
				10(3)			10(3)

(1) A $100 sale is shipped and the receivable is factored. Cash is received for 80% of the amount billed.

(2) A $50 payment is received from the customer and forwarded to the factor.

(3) The factor determines that 80% of eligible receivables held entitles COMPA to maximum credit of $40. COMPA draws $10.

To simplify representation of these transactions in the model, the same results are obtained by changing the sequence of entries. Again using "T" accounts to reflect the revised series of transactions:

Sales		Accounts Rec.		Cash		Factor	
	100(1)	100(1)	50(2)	80(1)	40(3)	40(3)	80(1)
				50(2)			

(1) A $100 sale is shipped and the receivable is factored. Cash is received for 80% of the amount billed.

(2) A $50 payment is received from a customer.

(3) The factor determines that 80% of eligible receivables ($50) only entitles COMPA to a $40 credit balance and COMPA is required to pay the factor $40.

Inclusion of these transactions in the model requires two special factoring constraints. The first represents debits and credits to the account and the second determines the maximum end of period balance that can be supported by eligible receivables. The cash flow constraint determines the amount of funds needed and provides cash to reduce the balance.

g.1: Transactions in the Factor Account:

The first special constraint has the following form:

$$FB\$_0 + x \sum_{t=1}^{T} SJ\$_t + x' \sum_{t=1}^{T} SC\$_t - \sum_{t=1}^{T} FP\$_t + I \sum_{t=1}^{T} FB\$_t = FB\$_T \qquad (7A)$$

or

$$-x \sum_{t=1}^{T} SJ\$_t - x' \sum_{t=1}^{T} SC\$_t + \sum_{t=1}^{T} FP\$_t - I \sum_{t=1}^{T-1} FB\$_t + (1.-I)FB\$_T = FB\$_0$$

$$T = 1,M$$

where:

x = The fraction of amount billed received by COMPA when Division 1's sales are factored. Slightly less than .8 because $SJ\$_T$ must be adjusted to allow for sales returns and allowance credits.

x' = Analogous to x, except applicable to Division 2's sales.

I = The monthly interest rate. For July and August, I = .01375, for September and October, I = .014.

This equation insures that the balance owed the factor at the end of each period is equal to the beginning balance, plus additional borrowing and interest accrued, less repayments. Because the factor actually accrues interest daily it is accrued in the model as a function of the current month's balance.

g.2. Maximum Factor Balance:

The second special factor equation has the form:

$$.8\,[(1.-e_T)SJ\$_T + (1.-e_T-e_{T-1})SJ\$_{T-1} + (1.-e_T-e_{T-1}-e_{T-2})SJ\$_{T-2}$$

$$+(1.-e_T')SC\$_T + (1.-e_T'-e_{T-1}')SC\$_{T-1} + (1.-e_T'-e_{T-1}'-e_{T-2}')SC\$_{T-2}] \geq FB\$_T$$

or (7B)

$$.8\,[(1.-e_T)SJ\$_T + (1.-e_T-e_{T-1})SJ\$_{T-1} + (1.-e_T-e_{T-1}-e_{T-2})SJ\$_{T-2}$$

$$+(1.-e_T')SC\$_T + (1.-e_T'-e_{T-1}')SC\$_{T-1} + (1.-e_T'-e_{T-1}'-e_{T-2}')SC\$_{T-2}]$$

$$-FB\$_T \geq 0$$

$$T = 1,M$$

where:

e_T, e_{T-1}, e_{T-2} = The proportion of Division 1's sales in periods T, T-1, and T-2 collected in period T.

e_T', e_{T-1}', e_{T-2}' = The proportion of Division 2's sales in periods T, T-1, and T-2 collected in period T.

This constraint determines the maximum end of period balance that can be supported by total eligible receivables outstanding. The maximum balance is used as a floating upper limit on the amount that can be owed the factor at the end of each period.

h. Cash Flow:

The COMPA cash flow restraint appears as follows:

$$R_M \sum_{t=1}^{T-1} RTM_t + w_M R_M(RTM_T) + R_S \sum_{t=1}^{T-1} RTS_t + w_S R_S(RTS_T) + R_C \sum_{t=1}^{T-1} RTC_t$$

$$+ w_C R_C(RTC_T) + R_B \sum_{t=1}^{T-1} RTB_t + w_B R_B(RTB_T) + R_M' \sum_{t=1}^{T-1} ETM_t + w_M R_M'(ETM_T)$$

$$+ R_S' \sum_{t=1}^{T-1} ETS_t + w_S R_S'(ETS_T) + R_C' \sum_{t=1}^{T-1} ETC_t + w_C R_C'(ETC_T) + R_B' \sum_{t=1}^{T-1} ETB_t$$

$$+ w_B R_B'(ETB_T) + \sum_{t=1}^{T} GUS_t + \sum_{t=1}^{T-1} RRM_t + K \sum_{t=1}^{T-1} WGJ_t + K' \sum_{t=1}^{T-1} WGC_t$$

$$+ m \sum_{t=1}^{T-1} SJ\$_t - x \sum_{t=1}^{T} SJ\$_t - x' \sum_{t=1}^{T} SC\$_t - (y)(e_T)SJ\$_T - e_T'(SC\$_T) - (y)(e_{T-1})SJ\$_{T-1}$$

$$- e_{T-1}'(SC\$_{T-1}) - (y)(e_{T-2})(SJ\$_{T-2}) - e_{T-2}'(SC\$_{T-2}) - (y)(e_{T-3})SJ\$_{T-3}$$

$$- e_{T-3}'(SC\$_{T-3}) - (y)(e_{T-4})SJ\$_{T-4} - e_{T-4}'(SC\$_{T-4}) + \sum_{t=1}^{T} FP\$_t - LB\$_T$$

$$+ CA\$_T = CS\$_0 - \sum_{t=1}^{T} NMR_t \qquad (8)$$

where: $T = 1,M$

w_i = The fraction of net cash wages earned in period T that is paid in period T. The subscript i indicates the labor process, M1L, SOL, C1L, or B1L.

R_i = Net cash wages per straight time manhour worked in process i.

R_i' = Net cash wages per overtime manhour worked in process i.

x, x' = See equation 7A.

y = The proportion of credits to Division 1 receivables that represent cash inflow. This adjustment is not applied to Division 2 receipts because it recognizes cash discounts taken by Division 1's customers, or for Division 2, $y' = 1$.

$e_T, e_{T-1}, e_{T-2}, e_{T-3}, e_{T-4}$ = The fraction of Division 1's sales in periods T, T–1, T–2, T–3, and T–4 collected in period T.

$e_T', e_{T-1}', e_{T-2}', e_{T-3}', e_{T-4}'$ = The fraction of Division 2's sales in periods T, T–1, T–2, T–3, and T–4 collected in period T.

m = The proportion of Division 1 sales paid to brokers as their commission. The value of "m" is less than the explicit rate of .05 because it must include an adjustment to convert sales to a net basis.

K = The freight out rate per pound paid by Division 1.

K' = The freight out rate per pound paid by Division 2.

NMR_t = All cash inflows and outflows during period t that are not related to variables in the matrix.

In the cash flow equation there are several explicit one period lags in addition to the adjustments that account for accrual of wages. These lags only represent actual trade credit terms in a general way. Payments for sugar (GUS) are not lagged because sugar is obtained on essentially C.O.D. terms.

Compliance with these terms is mandatory and comparison of the direct cost of sugar in the first ten months of 1966 with payments to suppliers indicates that the two quantities vary directly and proportionately. In addition, regressions of payments on total pounds produced and on credits to the sugar inventory account both had r^2 values above .9. In constast, the erratic pattern of payments to suppliers of other raw materials thwarted all attempts to develop either a single or multi-period distribution of payments that could be functionally related to model variables (RRM). For this reason, payments for all other raw materials were lagged one period. This does not represent compliance with n/30 trade credit terms because the variable RRM represents material used in production, not purchases.

Payments for freight out are also lagged one period. Although federal law requires all freight bills to be paid within seven days, most carriers tolerate payment within 30 days. The final lagged payment is the Division 1 brokers' commissions. The company usually pays a broker when his patience is completely exhausted. However, because the amounts involved are small and no pattern could be established for actual payments, commissions are assumed to be paid when due or in the month following the month of sale.

It should also be noted that interest payments are not explicitly recognized in the cash flow equation. Factor interest is accrued in equation (7A) and paid as a part of the total payment (FP$). No provision is made for interest payments for the bank overdraft (LB$). Although drawing against float may result in extra service charges for returned checks, the bank cannot levy an interest charge for a practice it does not condone.

i. Maximum Overdraft:

This constraint establishes an implicit upper limit on allowable overdraft. In equation (8), the variable LB$ is included to permit utilization of bank float as a source of funds to "unlock" the model. Because bank float is an actual source of funds it must be included in the model or cash will be inadequate to permit the test simulation. This inequality has the form:

$$LB\$_T \leqslant XLC_T \qquad (9)$$
$$T = 1,M$$

where:

XLC_T = Maximum amount of overdraft permitted in period T. An arbitrarily established parameter that has no counterpart in the firm.

j. Objective Functions:

The overall objective of COMPA's management is to maximize profit. To facilitate understanding of results and to provide a check on the internal consistency of the model, two functionals will be developed. The first objective function has the form:[1]

$$\text{maximize } \pi = \sum_{t=1}^{M} \sum_{j=1}^{N} gp_j S_{jt} - .5f_M \sum_{t=1}^{M} ETM_t - .5f_S \sum_{t=1}^{M} ETS_t$$
$$- .5f_C \sum_{t=1}^{M} ETC_t - .5f_B \sum_{t=1}^{M} ETB_t - K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t$$
$$- I \sum_{t=1}^{M} FB\$_t - I' \sum_{t=1}^{M} LB\$_t \qquad (10A)$$

where:

gp_j = Net contribution per pound of product j sold.

f_i = Regular time hourly pay scale in process i. The overtime premium is .5 of this rate.

K,K′ = See equation (8)

I = See equation (7A)

I′ = Implicit monthly interest rate applicable to the overdraft variable, LB$. This is an arbitrarily assigned parameter that has no counterpart in the firm.

As formulated, the functional (10A) will provide a maximum profit figure closely resembling conventionally defined gross margin. The coefficients, gp_j, approximate gross margin per pound of product j sold and because most production related costs are excluded, inventory adjustments are not required. The only direct cost reflected in the objective function is the premium paid for such hour of

[1] As formulated, the objective function maximizes because all profit coefficients are positive and all cost coefficients are negative. However, because the LP-90 computer code can only minimize, the functional must be multiplied by -1. to reverse all signs before the problem is submitted to the computer for solution. This is a technical manipulation that does not involve conceptual aspects of the model.

overtime worked. Inclusion of the premium reduces gross contribution by the extra cost attributable to overtime. This adjustment is required for two reasons. First, unit direct labor costs were calculated using straight time rates only. Second, because the cost of straight time was included in unit cost, it does not enter the functional. For this reason, the model considers straight time as costless and exhausts the hours available in all processes and periods, to the extent profitable, before overtime is used.

The other cost elements in the functional are in the "below the line" category. They represent freight costs and interest expense. Freight was included because it varies directly with sales but was not considered when individual product unit costs were calculated. Interest expense is of two types. The rate applied to the factor balance is an explicit cost supplied by the firm. No explicit rate applies to the overdraft and it is assigned an implicit rate to insure that sources of funds are utilized in the order desired. In the COMPA model $I' > I$ exhausts all credit available from the factor before the overdraft is used. This relationship limits the overdraft to its "unlocking" function and prevents it from superceding the factor as the primary source of external funds.

The second functional is formulated as follows:

$$\begin{aligned}\text{maximize } \pi = & \sum_{t=1}^{M}\sum_{j=1}^{N} nsp_j S_{jt} - f_M \sum_{t=1}^{M} RTM_t - f_S \sum_{t=1}^{M} RTS_t \\ & - f_C \sum_{t=1}^{M} RTC_t - f_B \sum_{t=1}^{M} RTB_t - 1.5_M \sum_{t=1}^{M} ETM_t - 1.5f_S \sum_{t=1}^{M} ETS_t \\ & - 1.5f_C \sum_{t=1}^{M} ETC_t - 1.5f_B \sum_{t=1}^{M} ETB_t - \sum_{t=1}^{M} GUS_t - \sum_{t=1}^{M} RRM_t \\ & - K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FBS_t - I' \sum_{t=1}^{M} LBS_t \end{aligned} \tag{10B}$$

where:

nsp_j = Net selling price per pound of product j.

f_i = See equation (10A). The overtime rate is 1.5 times the straight time rate.

Maximizing the functional in this form provides a profit figure that cannot be directly compared with any conventional measure. The positive term accumulates total sales revenue for the planning cycle. A figure equivalent to direct cost of goods manufactured during the same period is determined by the production related negative terms representing sugar, raw material and labor costs. The difference between the two quantities, gross revenue and cost of goods manufactured, is obviously not gross margin. To convert this difference to gross margin for the period requires that a supplemental adjustment be made to recognize any change in finished goods inventory that occurred. Terms for freight out and interest expense have the same interpretation in this expression as in (10A).

4.3.2 The Fully Integrated Simulation Model

Equations (1) through (10A) constitute the general form of the COMPA fully integrated linear programming model. The production related constraints [(2), (3), and (4)] were carried forward from the sub-model and values for their coefficients are shown in Appendix B. However, before the fully integrated model can be tested, specific values must also be determined to represent financial relationships that existed during the July–October 1966 test period.

Rather than develop the financial constraint coefficients sequentially it will save repetition to derive those with common properties by groups and relate them to the constraints in which they appear. The first such category includes selling prices and contribution for each product. This group contains "sp_j" that appears in equations (6A) and (6B), "gp_j" from (10A), and "nsp_j" that is used in (10B). The values of these coefficients for each of the 46 products are shown in Table 4.6. For products sold by Division 1, sp_j is list price per pound of net case weight. Net selling price (nsp_j) is 95% of list. This adjustment recognizes that 5% of each sales dollar must be paid to brokers and that the effective price is 95% of list price. Contribution (gp_j) is net revenue less total direct cost. For Division 2, sp_j is somewhat less than list price because gross sales must be averaged over total pounds shipped to allow for "free goods". For example, the list price of product "KB" is $.36 per pound.

Table 4.6
COMPA

Values for Coefficients
sp_j, nsp_j, and gp_j

(j)	(sp)	(nsp)	(gp)
JB	.350	.332000	.161370
JC	.45	.427000	.230750
JD	.455	.432	.242950
JF	.446	.424	.239250
JG	.467	.444	.237
JI	.450	.427	.243550
JJ	.462	.439	.240150
JL	.450	.427	.243450
JN	.518	.492	.243000
JO	.450	.427	.241750
JP	.432	.410	.229450
JR	.510	.485	.247450
JX	.433	.411	.121600
JY	.415	.394	.094700
SB	.320	.304000	.128350
SD	.405	.385000	.147800
SE	.405	.385000	.1677
SF	.405	.385000	.147300
SH	.355	.337500	.060500
SI	.355	.337500	.092800
SJ	.355	.337500	.080700
SL	.331	.314000	.07
SM	.363	.345000	.052900
SN	.379	.360000	.080600
*KB	.322	.318000	.145480
KC	.661	.628	.171740
KD	.531	.505	.314320
KE	.836	.795	.242590
KF	.732	.695	.153540
TA	.433	.412000	.15751
TC	.357	.339	.08691
TD	.391	.372	.09191
*BA	.291	.287000	.13807
*BB	.291	.287000	.14251
*BC	.299	.295000	.133440
*BD	.320	.316000	.146560
BF	.56	.532000	.224250
BG	.490	.445000	.148950
BI	.383	.364000	.110550
BJ	.420	.398000	.113750
*MA	.323	.318000	.0732
*MB	.243	.239500	.06803
MC	.468	.445000	.19667
MD	.412	.391000	.18306
*ME	.275	.271000	.08406
*MF	.284	.280000	.1112

* = Products sold by division 2

During the first nine months of 1966, Division 2 shipped 1,297,479 pounds of "KB" and recorded gross sales of $417,925. This is an average price of $.322 per pound as shown for "KB" in Table 4.5. All gross prices for Division 2's products were calculated in this manner. Net selling price (nsp_j) for products sold by Division 2 is estimated to be 98.6% of average gross price (sp_j). This adjustment recognizes miscellaneous trade and special discounts that make the effective price slightly less than the average gross price. Contribution (gp_j) is again defined as net revenue less total direct cost.

The coefficients "x" and "x'" are used in equations (7A) and (8) to make cash flow from the factor a function of gross dollar sales. This rate is given as .8. However, credits issued to customers for sales returns and allowances reduce the pool of eligible receivables and they must be offset by either new sales or repayments. For this reason, x and x' must also include an adjustment to change gross sales to a net basis. Although sales returns and allowances varied erratically from month to month during 1965 and 1966, average experience provides reasonably good estimates for x and x'. For Division 1, x = .8(1. – Sales R & A/Gross Sales) = .8(1. – .03) = .775. For Division 2, x' = .8(1. – .014) = .79. In addition, another coefficient "y" is used in equation (8) to eliminate cash discounts taken by Division 1's customers. The quantity remaining after this modification represents net cash remitted. Because most of Division 1's customers discount, y = .98.

The most difficult values to estimate were those for the coefficients $e_T, \ldots, e_{T-4}$ and $e_T', \ldots, e_{T-4}'$, that appear in equations (7B) and (8). These coefficients reflect customer payment patterns or the proportion of each month's sales collected in subsequent months. The necessary values could have been readily calculated from a standard aging of accounts receivable. However, COMPA aged receivables infrequently and it was necessary to estimate values for e_T and e_T' from data provided by the factor. During 1965 and 1966 the factor used cut-off dates for his reports that varied from the 15th to the 25th of each month and it was extremely difficult to relate available data to the calendar month accounting periods used by COMPA. For this reason the values established for e_T and e_T' are rather crude averages and it was impossible to isolate the seasonal payment patterns that appeared to be

present. Again in this instance, available data did not provide all the information needed although it seems that the model's data requirements are reasonable.

The values that will be used for e_T and e_T' are shown in Table 4.7. They have been tested using historical sales to predict end of the month accounts receivable balances and appear to be adequately representative.

Table 4.7
COMPA
Values for Coefficients e_T and e_T'

Division 1		Division 2	
e_T	.51	e_T'	.32
e_{T-1}	.30	e_{T-1}'	.29
e_{T-2}	.13	e_{T-2}'	.06
e_{T-3}	.04	e_{T-3}'	.21
e_{T-4}	.01	e_{T-4}'	.06

Coefficients that apply various direct labor cost factors to hours worked are used in equations (8), (10A), and (10B). As previously mentioned, direct unit cost includes all labor cost at straight time rates. However, higher cost overtime is also used and both straight time and overtime require cash to meet scheduled payrolls. In the functionals (10A) and (10B) the coefficient "f_i" represents the average hourly rate of pay for per diem workers in process i. These rates were supplied by COMPA and are shown in Table 4.8. The coefficients R_i and R_i' are used in equation (8) to reflect the net cash payment to employees for one hour of straight time or overtime worked in process i. Historical data indicates that about 82.5% of each dollar of direct, variable, labor cost is paid to employees and that 17.5% is withheld for various payroll deductions. Under current IRS regulations this should not be a particularly significant source of funds because most of the withholding is due the following month. In order to use withholdings as a source of funds, COMPA typically delays transmission to IRS and pays the

resulting interest and penalties. The values shown in Table 4.8 for R_i and R_i' were calculated as follows: $R_i = .825(f_i)$; $R_i' = .825\ (1.5f_i)$ . . .

The remaining labor related coefficient (w_i) is used in equation (8) to incorporate the wage accrual pattern inherent in the production process. This coefficient reflects the net effect of two company characteristics. First, that labor in the various processes is applied in a regular sequence during the production cycle. Second, that COMPA payroll procedures define a work week as Sunday through Saturday with payday the following Friday. Average experience indicates that the net effect of these two circumstances provides a source of funds that should be explicitly recognized in the cash flow equation. Estimated values for w_i are also shown in Table 4.8.

Table 4.8
COMPA
Values for Coefficients
f_i, R_i, R_i', and w_i

(i)	(f)	(R)	(R′)	(w)
M1L	$1.60	$1.32	$1.98	.93
SOL	1.60	1.32	1.98	.93
C1L	1.66	1.37	2.06	.82
B1L	1.50	1.24	1.86	.75

To complete the explanation of coefficient derivation, values are needed for "K", "K′" and "m" that appear in equations (8), (10A), and (10B). K and K′ are the average freight rates per pound of net finished weight shipped by Divisions 1 and 2 respectively. These rates were found by dividing total freight cost for each division by total net weight shipped. Calculated values are: K = .04 and K′ = .03. The coefficient "m" represents the fraction of Division 1's gross sales paid to brokers. Although the commission is nominally 5% of sales, brokers are penalized for returns and allowances

and the commission payment coefficient must be modified to a net sales basis, or m = .05(1. – Sales R & A/Gross Sales) = .05(1. – .03) = .05(.97) = .485.

The next step in preparing the general model to simulate the test period is to determine right hand side values for equations (1) through (8). Referring to the individual expressions it is noted that right hand side values for equations (3), (4), (5A), (5B), (6A) and (6B) are zero. Other equations only require that appropriate balances be found in conventional accounting records or reports. For example, in equation (1) the required right hand side values are the individual product finished goods inventory balances (net weight pounds) at the beginning of the simulation period (July 1, 1966). These quantities were found in the June 30, 1966 finished goods inventory report and are shown in Appendix C. Similarly, the right hand side value for equation (7A) is the amount owed the factor at the beginning of July (June 30, 1966), or $175,100, which was also readily ascertained from accounting records.

Equation (7B) appears to have a right hand side value of zero. However, in the first two periods it is necessary to provide the model with information concerning terms related to variables that precede the test period. For example, in July the variable $SJ\$_{T-2}$ refers to May sales which are unknown to the model. Using actual sales for each division for May and June, the right hand side of (7B) is calculated as follows:

$$\text{Period 1} = .8[(1. - e_T - e_{T-1})SJ\$_{T-1} + (1. - e_T - e_{T-1} - e_{T-2})$$
$$SJ\$_{T-2} + (1. - e_T' - e_{T-1}')SC\$_{T-1}$$
$$+ (1. - e_T' - e_{T-1}' - e_{T-2}')SC\$_{T-2}]$$

substituting previously determined values:

$$\text{Period 1} = .8[(.19)87{,}496 + (.06)96{,}935 + (.39)139{,}111$$
$$+ (.31)110{,}023] = .8(110{,}200) = 89{,}550$$

Using the same procedure the value for period 2 is found to be 39,800. Beginning with period 3 the right hand side is zero because all variables are within the test period. However,

when the values for periods 1 and 2 are transferred to the right hand side of (7B) they become negative.

The right hand side values for equation (2) are determined by multiplying daily supervisory manhours in each process by the number of working days in the period. The product of this multiplication is FT_{iT}. Values that are used for FT_{iT} is the simulation test are shown in Table 4.9.

Table 4.9
COMPA
Right Hand Side Values; FT_{iT}

		Period			
		(1)	(2)	(3)	(4)
Number of work days in the period:		20	23	21	20
(i)	Hours per day				
M1L	24	480	552	504	480
SOL	18	360	414	378	360
C1L	27	540	621	567	540
B1L	22	440	506	462	440

The cash flow constraint (8) requires the most complicated right hand side calculation. These values must include flows that are functions of variables that precede the planning period (as in 7B) and non-model related flows (MMR). In addition, as formulated in the multi-period model, all values are cumulative from the first period and the right hand side value for period T is the sum of all periods from 1 through T. Derivation of right hand side values for each period are shown in Table 4.10 and it should be noted that they are net figures. That is, gross flows are used in the calculations primarily because available data were aggregated and it is more efficient to eliminate estimates of cash flows that

Table 4.10
COMPA

Calculation of Right Hand Side Values for Equation (8)

Item	Period* (1)	(2)	(3)	(4)
Cash outflows:				
Misc. & freight	$ 46,762	$ 65,162	$ 63,815	$ 47,954
Trade accounts	36,452			
Div. 1 commissions	5,081			
Total net payroll	41,689	34,726	43,078	34,214
To subsidiaries	2,400		1,500	1,500
Federal withholding		21,246		
State withholding		2,212		
Initial cumulative cash deficit	44,671			
Total outflows	$177,055	$123,346	$108,393	$ 83,668
Inflows and adjustments:				
Freight to LHS	$	$ 19,035	$ 21,726	$ 21,304
[1]Direct portion, net payroll	11,250	13,600	17,450	14,555
Misc. receipts	1,581	1,626	1,377	623
From LHS	114,431	53,712	43,075	9,208
Total adjustments	$127,262	$ 87,973	$ 83,628	$ 45,690
Net outflow	$ 49,793	$ 35,373	$ 24,765	$ 37,978
Cumulative net outflow	$ 49,793	$ 85,166	$109,931	$147,909

* Periods are coded XCA_T in the initial tableau

1. The direct portion, net payroll is calculated as follows:

Direct payroll:				
Straight time	$ 17,276	$ 19,468	$ 26,411	$ 20,413
Overtime	2,152	2,819	2,070	3,040
	19,428	22,287	28,481	23,453
Less: Supr. payroll @$1,450 per week	5,800	5,800	7,250	5,800
Direct var. PR	$ 13,628	$ 16,487	$ 21,231	$ 17,653
Variable PR reduced to net flow basis; .825(DVPR)	$ 11,250	$ 13,600	$ 17,450	$ 14,555

will be functions of variables in the model than to directly aggregate non-model related flows. It should also be noted that the actual beginning overdraft is specifically deducted from the right hand side.[2] This forces the variable LB$ to pickup an overdraft amount to "unlock" the model that is directly comparable with COMPA's cash deficit.

With the addition of specific values for coefficients and right hand sides, the COMPA fully integrated model is ready to be tested. Using the model to simulate a period in the company's recent history, the testing hypothesis is: "If the model is forced to produce and sell the exact quantity and mix of items actually produced and sold during the test period, predicted levels of profit, sales revenue, borrowing, cash flow, etc., should approximate actual."

Proceeding with the simulation requires that all general parameters be defined in terms of the test limits. First, the test period will be the four months, July–October 1966, used to validate the production sub-model, or M = 4. However, this test will involve a multi-period or dynamic simulation rather than the static tests used for the sub-model. The dynamic model will also consider the full array of products, N = 46, including 37 items sold by Division 1 and 9 sold by Division 2, or L = 9. The implicit overdraft limit "XLC" in equation (9) is set at $100,000 to insure that the model will have adequate funds to preclude an infeasible solution. I' in the functionals (10A) and (10B) is assumed to be .02 to conform to the criterion $I' > I$. In addition, constraint equations must be added to the general model to provide information concerning actual production and sales levels and to force the simulation.

The first simulation constraint establishes the quantity of each product that must be produced in every period. It has the form:

$$P_{jT} = Y_{jT} \qquad \begin{matrix} T = 1,M \\ j = 1,N \end{matrix} \qquad (11)$$

[2] The amount of actual overdraft deducted in Table 4.10 is not the end of June credit balance in COMPA's cash account. Rather, it is the year to date net excess of cash payments over cash receipts. The book balance was not used because COMPA's practice of borrowing over the year end in order to show a positive cash balance for statement purposes tends to obscure the true cash position.

Where:

Y_{jT} = The quantity in pounds of product j produced during period T. Actual values for Y_{jT} are shown in Appendix C.

Next, a constraint is required to defined the quantity of each product sold in every period. This constraint appears as follows:

$$S_{jT} = D_{jT} \qquad T = 1,M \quad j = 1,N \tag{12}$$

Where:

D_{jT} = The quantity in pounds of product j sold during period T. Actual values for D_{jT} are shown in Appendix C.

The last special constraint limits the number of straight time hours available in each process to the amount actually worked during the test months. This constraint is formulated as an inequality because it provides a maximum supply but does not require that all available straight time hours be utilized. The inequality has the form:

$$RT_{iT} \leqslant XT_{iT} \qquad T = 1,M \quad i = \text{M1L, SOL, C1L, or B1L} \tag{13}$$

Where:

XT_{iT} = Maximum number of hours available at straight time pay rates in process i during period T. Actual straight time hours worked during July-October 1966 are shown in Table 5.11.

Table 4.11
COMPA
Right Hand Side Values; XT_{iT}
(hours)

	Period (T)			
(i)	(1)	(2)	(3)	(4)
M1L	930	1419	1078	942
SOL	1310	1906	1525	1471
C1L	2500	3239	2907	2721
B1L	4050	5181	4984	3983

The COMPA fully integrated, multi-period, linear programming model, constrained to simulate the test period, is now ready for solution. Because formulation of constraints and derivation of values has required a complicated and perhaps confusing discussion, the complete simulation model, consisting of equations (1) through (13), is summarized in Appendix D. Unfortunately the size of the model (626 rows and 624 vectors, excluding slack vectors) precludes presentation of the complete initial tableau. However, if the model is partitioned to eliminate the production related constraints and the simple inventory balance and special simulation constraints, the portion that contains the financial aspects of the fully integrated model is relatively small. This part of the initial tableau is shown in Appendix E.

4.3.3 Results of the Simulation Test

The model, as summarized in Appendix D, was solved using the LP-90 computer code.[3] This problem is large and after transformation to include slack vectors it generates a 626 X 648 matrix with 3,672 non-zero elements or a density of .905%. Because the problem is so big the complete solution output will not be shown. However, a portion of the solution, including values for all financial variables, is included as Appendix F.

In the solution format, all real and slack variables in the final basis are listed in the J(H) column. Each is identified by a code that conforms to the notation used in this chapter. For example, the first variable in Column J(H) is 1PJB. This code represents the variable P_{jT} for T = 1, j = JB. However, to facilitate data input the value of T is entered as a prefix rather than a subscript. Column BETA(H) shows the solution value for each variable listed in J(H). The ROW(I) column indicates equation codes or Row ID. Shadow prices or dual evaluators that apply to each row are shown in column PI(I) but they have very little meaning in this test and will not be discussed. Finally, column BI(I) repeats the right hand side values that were supplied as input.

[3] See CEIR, Inc., LP-90 User Manual for detailed instructions for preparation and format of input data.

The representativeness of the fully integrated model will be evaluated by comparing values from the simulation solution with appropriate actual values found in the records and accounts of COMPA. These comparisons, and percentage differences between each pair of values, are shown in Table 4.12. No specific criteria have been defined to determine if the model's financial estimates are acceptable. The decision to accept or reject individual estimates must be made subjectively considering the nature of the variable, its importance, the quantity of funds involved, and the size of the error. Obviously, because individual judgement is involved, opinions will differ. However, model values should not be viewed exclusively as point estimates but rather as mid-points or means of ranges of possible values. These ranges cannot be defined statistically but must be determined by the potential user of the model.

The actual/model comparisons in Table 4.12 are straight forward and generally self-explanatory. However, it may be useful to discuss some of the more important relationships. One of the most important comparisons contrasts actual with predicted contribution. In this case actual contribution cannot be taken directly from COMPA's records or statements. Rather, a quantity that can be compared with predicted contribution is determined as the net value of several components. Similarly, the objective function value (1PFT) cannot be used directly until the implicit interest charge on the total overdraft is eliminated. This correction must also be applied to the secondary functional (2NSP) in addition to the correction needed to reflect the net change in finished goods inventory. After comparable contribution figures have been derived, both primary (1PFT) and secondary (2NSP) predictions compare favorably with actual. This result strongly implies that the functionals are representative of the profit generating process and adequately reflect the important elements of revenue and cost.

In section (b) of Table 4.12, month-by-month comparisons of gross dollar sales indicates that the model provides estimates that closely approximate actual. This result is expected because information concerning the exact quantity for each product sold (pounds) was supplied by simulation equation (12).

Table 4.12
COMPA

Comparison of Financial Estimates Made by the Fully Integrated Model with Actual Experience during July-October 1966

a. Contribution:

Model estimates:

Contribution basis:		
Contribution (1PFT)		$ 266,214
Plus: Implicit interest		1,713
Adjusted contribution		$ 267,927
Percentage variation from actual		1.0
Revenue basis:		
Contribution (2NSP)		$ 285,555
Less: 6/30 F.G. inventory @ DC	27,650	
Direct supervision	24,650	52,300
		233,255
Plus: 10/31 F.G. Inventory @ DC	19,420	
Increase in W-in-P	6,103	
Implicit interest	1,713	27,236
		$ 260,491
Percentage variation from actual		(2.0)

Actual:

Gross sales, less div. 2 discounts		$ 830,381
Less: Direct labor	107,606	
Sugar	194,807	
Other raw materials	157,885	
Commissions	15,594	
Freight out	71,497	
Interest exp., factor	15,166	
6/30 F.G. inv. @ DC	27,650	590,205
		240,176
Plus: Increase in W-in-P	6,103	
10/31 F.G. inv. @ DC	19,420	25,523
		$ 265,699

Table 4.12 (continued)

	Actual	Model	Difference (%)
b. Gross sales:			
Div. 1: July (1SJ$)	$ 64,752	$ 65,206	.5
August (2SJ$)	106,852	107,214	.5
September (3SJ$)	99,005	98,969	.05
October (4SJ$)	72,914	72,935	.05
Total	$ 343,523	$ 344,324	.2
Div. 2: July (1SC$)	$ 127,102	$ 124,619	2.0
August (2SC$)	129,767	127,879	1.75
September (3SC$)	128,658	127,551	1.0
October (4SC$)	114,717	112,570	2.0
Total	$ 500,244	$ 492,619	1.5
Total, COMPA	$ 843,767	$ 836,943	.7
c. Factor account:			
Balance, EOM:			
July (1FB$)	$ 191,265	$ 182,779	4.5
August (2FB$)	214,916	200,059	7.0
September (3FB$)	198,464	199,283	.2
October (4FB$)	176,325	182,439	3.4
Total	$ 780,970	$ 765,560	2.0
Average balance	$ 195,242	$ 191,390	2.0
Net cash inflow:			
July	$ 202,600	$ 192,191	5.0
August	216,200	218,673	1.0
September	209,700	214,599	2.2
October	178,500	179,010	.3
Total	$ 807,000	$ 804,473	.3
Average monthly inflow	$ 201,750	$ 201,118	.3
d. Implicit overdraft:			
Balance. EOM:			
June	$ 44,671	$ 44,671	-
July (1LB$)	18,114	24,857	37.0
August (2LB$)	26,182	24,158	8.0
September (3LB$)	41,538	16,944	59.0
October (4LB$)	21,753	19,701	9.5

Table 4.12 (continued)

e. Cash payments:			
July: Misc. & freight	$ 46,762	$ 46,762	
Trade accounts	36,452	36,452	
Sales commissions	5,081	5,081	
To subsidiaries	2,400	2,400	
Sugar (1GUS)	45,269	42,428	
Payroll: Total	41,689		
Indirect & office		30,439	
Direct		11,886	
	$ 177,653	$ 175,448	
Less: Misc. receipts	1,581	1,581	
Total	$ 176,072	$ 173,867	1.5
August: Miscellaneous	$ 65,162	$ 46,127	
Freight		17,790	
Trade accounts (1RRM)	38,540	35,699	
Sales commissions	7,546	3,150	
Sugar (2GUS)	56,462	55,328	
Fed. withholding	21,246	21,246	
State withholding	2,212	2,212	
Payroll: Total	34,726		
Indirect & office		21,126	
Direct		16,357	
	$ 225,894	$ 219,035	
Less: Misc. receipts	1,626	1,626	
Total	$ 224,268	$ 217,409	3.5
Sept.: Miscellaneous	$ 63,815	$ 42,089	
Freight		21,755	
Trade accounts (2RRM)	57,660	45,944	
Sales Commissions	6,306	5,190	
To subsidiaries	1,500	1,500	
Sugar (3GUS)	54,096	49,842	
Payroll: Total	43,078		
Indirect & office		25,628	
Direct		16,383	
	$ 226,455	$ 208,331	
Less: Misc. receipts	1,377	1,377	
Total	$ 225,078	$ 206,954	8.0
Oct.: Miscellaneous	$ 47,954	$ 26,650	
Freight		21,310	
Trade accounts (3RRM)	32,954	43,984	
Sales commissions	25	4,770	
To subsidiaries	1,500	1,500	
Sugar (4GUS)	44,300	48,210	
Payroll: Total	34,214		
Indirect & office		19,659	
Direct		15,686	
	$ 160,947	$ 181,769	
Less: Misc. receipts	623	623	
Total	$ 160,324	$ 181,146	11.3

However, test results do establish that the coefficients "sp_j" satisfactorily convert pound sales by product into total divisional dollar sales.

The end of month balance owed the factor varies somewhat more than sales or contribution. This variation reflects the model's inability to exactly simulate the payment practices of the firm. In the model, cash flows must be linear functions of decision variables. If this functional relationship is tenuous, flows are distorted and ultimately all variations are reflected in total funds demanded. However, over the planning cycle monthly variations are smoothed and comparing total and average balances indicated a good overall representation of borrowing from the factor.

Cash flow from the factor was calculated outside the model. The first two terms in equation (7A) that measure gross cash flow from the factor were extended for each month and net repayments ($FP\$_T$) were subtracted. This measure of flow is compared with actual bank transfers recorded during the same period. Again monthly variations are apparent although none exceed 5%. Considering that these flows are subject to the same vagaries that afflict the factor balance, the model estimates appear adequately representative.

The largest percentage variations appear in the overdraft comparisons. Although these large differences are undesirable they are also understandable. The overdraft is intended to provide marginal funds to "unlock" the model when the factor balance upper limit becomes effective. For this reason, the absolute amounts involved do not appear excessive when evaluated in the context of total demand for external funds.

Proceeding from general to more specific comparisons, cash payments for each month are disaggregated in section (e) of Table 4.12. July payments have little opportunity to deviate because values for all lagged variables were supplied. In subsequent months only sugar and payroll estimates are consistently close to target. Payments to trade creditors and brokers are especially subject to wide and erratic variations and reflect the arbitrary one period lags used in the model. However, many of the larger variations can be explained. For example, in September extraordinarily large payments were

made to trade creditors. This caused the model estimate of "normal" payments to be considerably less than actual. The following month only mandatory payments were made to creidtors and brokers in order to accumulate funds to cover federal and state withholding payments that had to be made in early November. This tactic whipsawed the model and caused its estimate for October to be quite high.

The cash payment comparisons point out several areas where predictions did not adequately represent actual experience. However, for planning purposes the analytical shortcomings that caused these variations should not invalidate the model. Traditionally budgets and forecasts incorporate the assumption that average relationships will remain valid. The model uses this basic budgeting concept but it also permits the power of linear programming to be brought to bear on the total planning analysis.

Referring to the test hypothesis, that predicted levels of profit, revenue, cash flow, borrowing, etc., should approximate actual, the comparisons made in Table 4.12 indicate that the estimates made by the COMPA fully integrated simulation model reasonably represent actual experience during the test period. This conclusion precedes and logically leads to the finding that the model is adequately representative of COMPA and can be used for planning purposes.

The theoretical applications of linear programming to financial problems that were discussed in Chapters 1 and 2 have been subjected to their first and most difficult test. The fact that the most important aspects of COMPA's physical, financial and organizational structure could be successfully reduced to a series of mathematical expressions indicates that fully integrated financial models can be formulated to represent real firms. This finding supports the conclusion that, if the economic characteristics of a firm satisfy the linear programming criteria, it may be possible to formulate a fully integrated, multi-period model from data available in the firm. This statement is qualified because all necessary data may not always be available. For example, in COMPA, estimates for most coefficients and parameters were developed as a part of this research and are in addition to information regularly accumulated

or used by management. In some cases, basic data, e.g., capacity and labor consumption factors, had to be completely developed. Other information was located in company records but required considerable modification before it could be used.

4.4 The Aggregate Models

The COMPA fully integrated simulation model is quite large. It uses 626 of the maximum 1,025 rows that can be accommodated by the LP-90 computer code and if more periods are included in the planning cycle the model will soon exceed the capacity of the program. In addition, minor modifications to linear programming models of this size entail considerable clerical work and extensive punch card manipulation. Considering these problems it appears highly desirable to reduce the size of the COMPA model as much as possible in order to enhance its utility.

One of the decisions made before the COMPA model was formulated determined the number of products that would be represented. At that point over one-half of the product line was eliminated by aggregating items with identical cost and physical characteristics. To further reduce the number of products will require that physical similarities be matched and average unit cost and factor consumption coefficients determined to represent the common properties of each product sub-group.

4.4.1 Defining the Aggregate Products

There are two practical ways that the products listed in Table 4.1 can be aggregated. The first possible system of classification identifies each item with the division responsible for its sales. This is a feasible procedure although it seems unlikely that two products will provide the model with realistic alternatives. The second method combines individual items into six groups that conform to those shown in Table 4.1. However, grouping solely by physical characteristics is not practical because it eliminates fundamental divisional differences that must be recognized in the fully integrated model. For this reason it is necessary to divide groups "K", "B", and "M" into sub-groups by division. These additional categories permit recognition of both divisional and physical differences.

To complete the second set of aggregate products one additional classification must be included under Division 1. Items "SH", "SI", and "SJ" are low margin, high volume, seasonal specials that are not homogeneous with the other products in group "S". Combining these items to form a separate sub-group will facilitate calculation of better average unit cost and resource consumption factors for both "S" categories. For identification purposes the model based on two products will be called Aggregate Model 01 and the one that includes ten products will be Aggregate Model 02. Revised product groups, sub-groups, and codes that will be used in both aggregate models are summarized in table 4.13.

Table 4.13
COMPA
Aggregate Product Groups

	Product Group Code	Sales Div.	Individual Products Included	
			Groups	Items
Aggregate Model 01:				
	JT	1	J,S,T,	KC,KD,KE,KF, BF,BG,BI,BJ,MC,MD
	CG	2	–	KB,BA,BB,BC,BD, MA,MB,ME,MF
Aggregate Model 02:				
	JT	1	J	–
	ST	1		SB,SD,SE,SF,SL, SM,SN
	SJ	1		SH,SI,SJ
	KB	2		KB
	KT	1		KC,KD,KE,KF
	TT	1	T	–
	BC	2		BA,BB,BC,BD
	BT	1		BF,BG,BI,BJ
	MC	2		MA,MB,BE,MF
	MT	1		MC,MD

Before individual products are replaced by aggregate products, unit cost, margin and factor consumption coefficients have to be determined for each group. Values for these coefficients that will be valid over the range of possible production and sales volumes must recognize both individual product characteristics and average product mix. In Figure 4.1 it was shown that regressions of model estimates on total output closely approximated those based on actual values. However, those regression analyses used total cost, factor consumption and production for each period. If similar analyses were available in which the independent variables reflected output of individual products, the linear regression equations would predict cost or factor consumption as a function of each product's volume of output. These regression equations, in the form $Y = a + bX$, could then be used as coefficients in the linear programming matrix to functionally relate the consumption of resource Y to the production of X. To illustrate this point, Figure 4.2 shows actual and model regression lines similar to those plotted in Figure 4.1. Assuming only two products, if the quantities of Y used to produce known amounts of each product, during the periods included in the analysis, are determinable they can be used as dependent variables and regressed on the amounts produced. The resulting hypothetical regression equations, $Y_1 = a' + b'X_1$ and $Y_2 = a'' + b''X_2$ can be used to predict consumption of Y as a function of either X_1 or X_2 as illustrated in Figure 4.3.

Because the disaggregate equations are derived from the same data that was used as the basis for the total consumption equation, $Y = a + bX$, it follows that: $[(a' + b'X_1) + (a'' + b''X_2)] \cong (a + bX)$ or, $Y_1 + Y_2 = Y$. This additive property shows that if an element of cost or factor consumption is functionally related to output by product group, total consumption predicted for all products will approximate comparable aggregate experience.

Substituting a regression equation, $Y = a + bX$, for a single valued coefficient in a conventional linear programming constraint produces an expression with the following form.

$$\sum_{j=1}^{N} (a_{ij} + b_{ij}X_{jT}) = Y_{iT} \qquad T = 1,M$$

Figure 4.2
COMPA

Illustrative Aggregate
Linear Regression

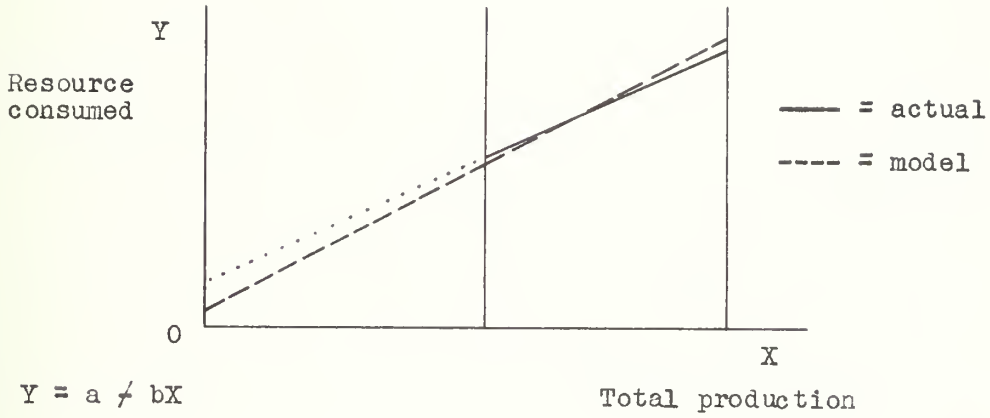
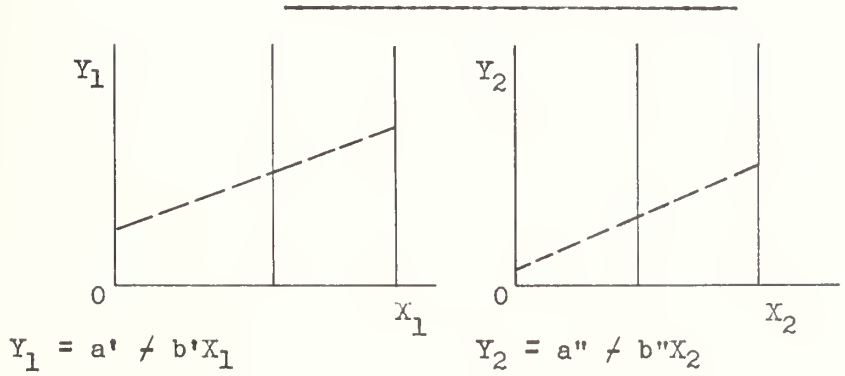


Figure 4.3
COMPA

Illustrative Disaggregate
Linear Regressions



or,

$$\sum_{j=1}^{N} b_{ij}X_{jT} - Y_{iT} = -\sum_{j=1}^{N} a_{ij}$$

When this expression is used in the linear programming matrix, the regression coefficient b_{ij} becomes the coefficient in the i row, X_j vector and the algebraic sum of the constants is transferred to the right-hand side. The amount of resource used is accumulated in the Y_{iT} variable as it would be in the regression equation, $Y = a + bX$. . .

4.4.2 Determining the Aggregate Coefficient Values

If the simple linear regression technique is to be used to determine the values of coefficients that apply to aggregate products, two pieces of information are required for each product in every period. First, the total amount produced and sold must be determined. This information is readily calculated by summing individual product output and sales volumes (as shown in Chapter 4, Appendix C) for each group. Second, total direct cost by element, total revenue, total margin and total factor consumption by resource must be quantified. These values become the dependent variables in the regression analysis. The production and sales volumes are the independent variables.

No data are available that indicate the proportion of actual resource consumption attributable to individual products. However, information from the COMPA model can be used to determine values for the dependent variables. These values are calculated by partitioning each row vector of product related coefficients in the fully integrated model into groups that conform to the two sets of aggregate products. Column vectors listing quantities of each product produced and sold in the various periods are also divided into aggregate product sets. Using the computer program written to determine model estimates for the production sub-model, each row vector for process i, product group j, is multiplied by the column vector for group j, period T. The inner products of these vector multiplications provide estimates of the quantity of resource i consumed in the production of product group j during period T,

or the dependent variable values needed for the regression analysis. For example, Table 4.14 indicates how the direct cost of sugar used to produce 19,805 pounds of product "BT" during July 1966 was estimated.

Table 4.14
COMPA
Direct Cost of Sugar, Product "BT"
July 1966

Individual products in the "BT" group	Cost Factor[1]	Quantity Produced[2]	Total Cost
BF	.09	10,688	$ 961.92
BG	.09	3,485	313.65
BI	.09	4,864	437,76
BJ	.09	768	69.12
Totals		19,805	$1,782.45

[1]See Chapter 4, Appendix B, cost row SUG
[2]See Chapter 4, Appendix C

For production related estimates, data were available for six months (February, July, August, September, October, and November 1966) and dependent variables values were calculated for each period. This permitted inclusion of six observations in the regression analyses that determine values for these variables. The analyses that use sales volume as the independent variable are based on dependent variable estimates for four months (July, August, September and October 1966). The values of the independent variables for each product group and dependent variable estimates for each cost element and resource are shown in Appendix G to this chapter. The linear regression equations based on these values are shown in Appendix H.

Referring to Appendix G it should be noted that values are not estimated for all possible product/resource combinations. Those omitted are aggregate products that have the same basic coefficient for all items in the group, e.g., sugar cost per pound for group "JT". In Appendix H coefficients of determination are indicated for each regression equation. Although most r^2 values are quite high, a few

are too low to inspire confidence in the estimates that can be made using a particular equation, especially since the regression analyses include only four or six observations. However, r^2 values for all high volume product groups are acceptable and, because somewhat less accurate estimates for low volume groups will not cause any systematic overall distortions, all necessary regression coefficients can be used.

The final area of concern caused by using regression coefficients in the linear programming matrix involves possible changes in relative desirability among products. This problem arises because a few of the equations have quite large constant terms. For example, if the basic coefficients for two products, j_1 and j_2, are b_1 and b_2 respectively, and $b_1 > b_2$, estimating values for b_1 and b_2 by regression analysis may produce equations in which $a_1 > a_2$ but $b_1 < b_2$. This result would reverse the order of trade-off between the two products. A careful check of the regression coefficients in Appendix H indicates that reversals of this sort should not be a problem in Model 02 although the gross aggregation in 01 obviously obscures product differences. In addition, the largest constant terms fortuitously occur in high volume products. This makes the average amount of constant per pound minimal. For example, the largest constant term that appears in Model 02 (Appendix H) is in equation (nsp) for product "BC", a = $7,617.40. The lowest monthly sales of "BC" during the July–October planning cycle are 198,342 pounds or a maximum average constant of $.038 per pound.

4.4.3 Modifying the Simulation Model

Previous sections of this chapter have explained the product aggregation rationale and the mechanics of coefficient determination by linear regression analysis. Substituting aggregate products in the COMPA simulation model is straight-forward and in most instances only involves including the regression constants and changing the limit "N" in the product summation terms. However, one important and unorthodox modification is required to permit revised product related coefficients to be used in the functional. Because the functional must have an initial right-hand side value of zero,

it is necessary to include constraints in the model to accumulate gross margin and sales revenue for the functionals. This procedure, in effect, allows a non-zero right-hand side value to be used in a functional. The first new equation accumulates gross margin and has the form:

$$\sum_{j=1}^{N} (Agp_j + gp_jS_{jT}) = GP\$_T$$

or (10C)

$$\sum_{j=1}^{N} gp_jSj_T - GP\$_T = -\sum_{j=1}^{N} Agp_j \qquad T = 1,M$$

Where:

Agp_j = The regression constant determined when gross margin for product j was regressed on sales.

gp_j = The regression coefficient determined when gross margin for project j was regressed on sales.

$GP\$_T$ = Total gross margin predicted for period T. This variable becomes the positive term in the functional (10A).

Including equation (10C) in the model also requires that the functional (10A) be revised to incorporate the variable $GP\$_T$. The revised functional has the form:

$$\text{maximize } \pi = \sum_{t=1}^{M} GP\$_t - .5f_M \sum_{t=1}^{M} ETM_t - .5f_S \sum_{t=1}^{M} ETS_t - .5f_C \sum_{t=1}^{M} ETC_t$$

$$- .5f_B \sum_{t=1}^{M} ETB_t - K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FB\$_t \qquad (10A\text{-}1)$$

$$- I' \sum_{t=1}^{M} LB\$_t$$

The second additional constraint accumulates sales revenue.

$$\sum_{j=1}^{N} (Ansp_j + nsp_jS_{jT}) = SP\$_T$$

or (10D)

$$\sum_{j=1}^{N} nsp_jT - SP\$_T = -\sum_{j=1}^{N} Ansp_j \qquad T = 1,M$$

Where:

$Ansp_j$ = The regression constant determined when sales revenue for product j was regressed on sales.

nsp_j = The regression coefficient determined when sales revenue for product j was regressed on sales.

$SP\$_T$ = Total sales revenue predicted for period T. This variable becomes the positive term in the functional (10B).

Including equation (10D) in the model also requires that the functional (10B) be revised to incorporate the variable $SP\$_T$. The revised functional has the form:

$$\text{maximize } \pi = \sum_{t=1}^{M} SP\$_t - f_M \sum_{t=1}^{M} RTM_t - f_S \sum_{t=1}^{M} RTS_t - f_C \sum_{t=1}^{M} RTC_t$$

$$- f_B \sum_{t=1}^{M} RTB_t - 1.5f_M \sum_{t=1}^{M} ETM_t - 1.5f_S \sum_{t=1}^{M} ETS_t \qquad \text{(10B-1)}$$

$$- 1.5f_C \sum_{t=1}^{M} ETC_t - 1.5f_B \sum_{t=1}^{M} ETB_t - \sum_{t=1}^{M} GUS_t - \sum_{t=1}^{M} RRM_t$$

$$- K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FB\$_t - I' \sum_{t=1}^{M} LB\$_t$$

4.4.4 Validating the Aggregate Models

To ascertain if the aggregated models are representative of the firm they will be tested by simulation identical to the procedure used to test the comprehensive model. However, because the models to be validated were aggregated from the comprehensive model, test estimates will be compared with model estimates, shown in Table 4.12, rather than actual experience. The test hypothesis is: "Because it has been established that the COMPA comprehensive fully integrated model is representative of the firm, if all estimates made by an aggregate model approximate comparable estimates from the fully integrated model, it will be concluded that the aggregate model is adequately representative of actual experience to be used for planning purposes."

Proceeding with the simulation tests requires that revised right-hand side values be calculated for each of the product related constraints. This is a straight-forward procedure that modifies the right-hand side values to reflect either the aggregation of products (equations (1), (11), and (12)), or the inclusion of regression constants (equations (2), (3), (4), (6A), (6B), (10C) and (10D)). For example, calculation of the revised right-hand side value for row 1SUG, that will be used in Aggregate Model 02, is shown in Table 4.15. All right-hand sides were revised in this manner and values that will be used in both aggregate model simulation tests are shown in Appendix I.

Table 4.15
COMPA
Aggregate Model 02
Calculation of Revised Right-Hand Side Value

Row: 1SUG		
	LHS	RHS
RHS value in comprehensive model		0
Regression constants for products:		
JT	0	
ST	0	
SJ	0	
KB	0	
KT	0	
TT	0	
BC	505.	
BT	0	
MC	183.	
MT	- 1.	
Total constant	687.	
To transfer constant to RHS	-687.	-687.
Revised value for 02		-687.

The final revision necessary before the aggregate models can be solved requires that summation limits "N" (total products) and "L" (number of products sold by Division 2) be restated to conform to the number of product groups to be included in each model. In Aggregate Model 01, N = 2,

L = 1, and in Aggregate Model 02, N = 10, L = 3. The test period will again represent July–October 1966 or M = 4 as in the simulation test of the comprehensive model.

4.4.5 Results of the Simulation Tests

The COMPA aggregate fully integrated models were solved using the LP-90 computer code. Model 01 used 106 rows and 128 vectors and had 556 non-zero elements or a density of 3.949%. Model 02 required 202 rows and 224 vectors and had 1,124 non-zero elements or a density of 2.436%. Both solutions reflect significant decreases in model size. If either one or both of the aggregate models can be used for planning purposes the COMPA short-term planning horizon can be extended considerably beyond four periods without exceeding the capacity of the LP-90 program.

The computer solutions for the 01 and 02 simulations are included as Appendixes J and K. Solution format is identical to that in Appendix F. To evaluate the results of these simulation tests, estimates made by each aggregate model are compared with those shown in Table 4.12 for the comprehensive model. These comparisons are reflected in Table 4.16. By inspection it is apparent that comparable values for the three models are essentially identical and that the test hypothesis is confirmed. This finding indicates that either aggregate model is adequately representative of the comprehensive model to be used as its proxy for planning purposes.

Before proceeding to the "what if" problems and other tests that require the model to optimize rather than simulate, a decision must be made concerning which of the three fully integrated models will be used. The limitations imposed by the size of the comprehensive model were discussed early in this chapter. These limitations motivated development of the aggregate models and consequently eliminate the comprehensive model from consideration since useable aggregate models are available. Aggregate Model 01, based on only two product groups, does not provide enough alternatives to permit realistic optimization when the simulation constraints are relaxed. Profit, revenue, and cash flow characteristics of the Division 1 product group (JT) are all superior to those for the Division 2

Table 4.16
COMPA

Comparison of Financial Estimates made by the Aggregate Models with those made by the Comprehensive Model

	Comprehensive Model	Aggregate Models: 01	02
Contribution:			
Contribution basis:			
Contribution (1PFT)	$ 266,214	$ 266,343	$ 266,285
Plus: Implicit interest	1,713	1,880	1,691
Adjusted contribution	$ 267,927	$ 268,223	$ 267,967
Revenue basis:			
Contribution (2NSP)	$ 285,555	$ 287,666	$ 287,407
Less: 6/30 F.G. inv.	27,650	27,650	27,650
Direct supr.	24,650	24,650	24,650
	233,255	235,366	235,107
Plus: 10/31 F.G. inv.	19,420	19,420	19,420
Increase in WIP	6,103	6,103	6,103
Implicit interest	1,713	1,880	1,691
Adjusted contribution	$ 260,491	$ 262,769	$ 262,321
Gross sales:			
Div. 1: July (1SJ$)	$ 65,206	$ 63,032	$ 65,073
August (2SJ$)	107,214	105,583	106,482
Sept. (3SJ$)	98,969	99,825	99,898
Oct. (4SJ$)	72,935	75,883	72,870
Total	$ 344,324	$ 344,323	$ 344,323
Div. 2: July (1SC$)	$ 124,619	$ 124,627	$ 124,588
August (2SC$)	127,879	127,299	127,618
Sept. (3SC$)	127,551	128,093	127,784
Oct. (4SC$)	112,570	112,597	112,627
Total	$ 492,619	$ 492,616	$ 492,617
Total, COMPA	$ 836,943	$ 836,939	$ 836,940

Table 4.16 (continued)

Factor account:			
Balance: EOM,			
July (1FB$)	$ 182,779	$ 181,931	$ 182,710
August (2FB$)	200,059	198,776	199,600
Sept. (3FB$)	199,283	199,381	199,567
October (4FB$)	182,439	183,681	182,556
Total	$ 765,560	$ 763,769	$ 764,433
Net cash inflow:			
July	$ 192,191	$ 190,155	$ 191,873
August	218,673	216,431	217,166
September	214,599	214,762	215,247
October	179,010	181,000	180,502
Total	$ 804,473	$ 802,347	$ 804,788
Implicit overdraft:			
Balance: EOM,			
July (1LB$)	$ 24,857	$ 27,288	$ 25,480
August (2LB$)	24,158	27,638	24,320
Sept. (3LB$)	16,944	20,211	16,841
Oct. (4LB$)	19,701	18,839	17,934
Total	$ 85,660	$ 93,976	$ 84,575
Cash payments:			
July:			
Miscellaneous	$ 46,762	$ 46,762	$ 46,762
Freight	36,452	36,452	36,452
Trade accounts	5,081	5,081	5,081
Sales commissions	2,400	2,400	2,400
To subsidiaries	42,428	42,893	42,746
Sugar (1GUS)	30,439	30,439	30,439
PR: Indirect & office	11,886	11,600	11,582
Direct	175,448	175,627	175,462
	1,581	1,581	1,581
Less: Misc. receipts	$ 173,867	$ 174,046	$ 173,881
Total			
August:			
Miscellaneous	$ 46,127	$ 46,127	$ 46,127
Freight	17,790	17,790	17,790
Trade accounts (1RRM)	35,699	35,219	34,899
Sales commissions	3,150	3,150	3,150
Sugar (2GUS)	55,328	55,120	54,920
Federal withholding	21,246	21,246	21,246
State withholding	2,212	2,212	2,212
PR: Indirect & office	21,126	21,126	21,126
Direct	16,357	16,134	16,187
	219,035	218,124	217,657
Less: Misc. receipts	1,626	1,626	1,626
Total	$ 217,409	$ 216,498	$ 216,031

Table 4.16 (continued)

September:			
Miscellaneous	$ 42,089	$ 42,089	$ 42,089
Freight	21,755	21,755	21,755
Trade accounts (2RRM)	45,944	46,572	45,959
Sales commissions	5,190	5,190	5,190
To subsidiaries	1,500	1,500	1,500
Sugar (3GUS)	49,842	50,285	50,598
PR: Indirect & office	25,628	25,628	25,628
Direct	16,383	16,235	16,351
	208,331	209,254	209,070
Less: Misc. receipts	1,377	1,377	1,377
Total	$ 206,954	$ 207,877	$ 207,693
October:			
Miscellaneous	$ 26,650	$ 26,650	$ 26,650
Freight	21,310	21,310	21,310
Trade accounts (3RRM)	43,984	43,257	42,906
Sales commissions	4,770	4,770	4,770
To subsidiaries	1,500	1,500	1,500
Sugar (4GUS)	48,210	47,929	47,708
PR: Indirect & office	19,659	19,659	19,659
Direct	15,686	15,205	15,601
	181,769	180,280	180,104
Less: Misc. receipts	623	623	623
Total	$ 181,146	$ 179,657	$ 179,481

Chapter 4
Appendix A
COMPA

Consolidated Operating Statements
Years ended December 31,

	1966	1965	1966
Gross sales	$3,044,775	$3,067,766	$3,045,547
Less: Returns and allowances	48,639	40,228	32,182
Net sales	2,996,136	3,027,538	3,013,365
Cost of goods sold	2,080,922	2,087,224	2,087,651
Gross profit	915,214	940,314	925,714
Cost of operations:			
Selling expense	296,934	377,906	384,914
Freight	234,961	246,331	252,714
Administration	179,830	174,560	177,336
Total	711,725	798,797	814,964
Operating profit	203,489	141,514	110,750
Other charges (net)	117,991	107,502	107,188
Net earnings	$ 85,498	$ 34,012	$ 3,562
Depreciation charged to operations	$ 79,102	$ 77,002	$ 75,039

Chapter 4
Appendix A (continued)
COMPA

Consolidated Balance Sheets
December 31,

	1966	1965	1964
ASSETS			
Cash[1]	$ 26,292	$ 48,122	$ 98,026
Accounts rec. (net)	336,424	339,372	307,305
Inventories	401,187	350,812	310,885
Fixed assets (net)	884,420	868,206	852,199
Miscellaneous	38,783	35,353	39,998
Total assets	$1,687,106	$1,641,865	$1,608,413
LIABILITIES			
Notes and accts pay.	$ 929,286	$ 979,744	$ 908,772
Accrued expenses	166,014	133,267	122,194
Current portion LTD	49,545	49,545	49,545
Deferred payables	30,814	38,357	49,510
Mortgages and LTD	166,776	181,819	232,423
5% debentures	173,643	173,643	194,768
Capital stock, no par	1	1	1
Retained earnings*	171,027	85,489	51,200
Total liab. and cap.	$1,687,106	$1,641,865	$1,608,413
*Capital surplus	$2,013,113	$2,013,113	$2,013,113
Less: Earned deficit	1,842,086	1,927,624	1,961,913
Retained earnings	$ 171,027	$ 85,489	$ 51,200

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[1] In order to show a positive cash balance on December 31st of each year, COMPA has been required to borrow at least $50,000. However, the bank requires that this loan be repaid immediately after New Years.

Chapter 4
Appendix B
COMPA

Initial Tableau Coefficients
Production Sub-Model

EQUATION CODES		PRODUCTS: JB	JC	JD	JF	JG	JI	JJ	JL	JN	JO	JP	JR	JX	JY	SB	SD	SE	SF	SH	SI	SJ	SL	SM	SN	KB	KC	KD	KE	KF	TA	TC	TD	BA	BB	BC	BD	BF	BG	BI	BJ	MA	MB	MC	MD	ME	MF
Process:	M1C	.1175														.095										.115								.11	.0925	.123	.105	.132	.132	.132	.132	.178	.147	.108	.1115		.065
	M1L	.0027														.00219										.0031								.0026	.0022	.00292	.00318	.00398	.00398	.00398	.00398	.00395	.00348	.00373	.00257		.00149
	S1C																																	.506	.423	.565	.655	.822	.822	.822	.822		.675				
	S2C	.758														.613										.825																		1.76	.72		.42
	S0L	.0045														.00356										.00385								.00243	.00207	.00273	.0035	.00396	.00396	.00396	.00396		.00325	.00625	.0042		.00243
	C1C	1.0														1.08										1.055					.981	.981	.981	1.55	1.55	1.1	1.295	1.085	1.085	1.085	1.085		.95	1.055	1.267	.854	1.67
	C1L	.00625														.00677										.00661					.00614	.00614	.00614	.0097	.0097	.0069	.00812	.00679	.00679	.00679	.00679	.00377	.00595	.00661	.0079	.0115	.01825
	P1C																																									1.0					
	P2C																								1.0																							
	P3C													1.0	.935																																	
	P4C																																					.641	1.0	1.0	1.0						
	P5C																																										1.0					
	P6C																														1.0	1.0	1.0															
	P7C																										1.0																					
	B1C																1.0		1.0	1.0		1.0	1.87	1.87					5.72		2.0	2.0							2.0	2.0			1.9	1.08			
	B2C																	1.0			1.0																											
	B3C	1.0															1.385		1.385	1.385		1.385	1.385	1.385																								
	B4C																									1.0		1.0	1.0					.735	.735	.818	.818										
	B1L	.00422	.01											.0334	.0383	.007	.012	.009	.0105	.0115	.0115	.0115	.0164	.0164	.0238	.00485	.1457	.00485	.00485	.05	.0354	.0353	.0353	.00196	.00196	.00262	.00262	.0074	.0522	.0379	.0379	.0334	.0182	.0163	.0108	.019	.00333
Cost:	DLC	.03651	.04188											.07303	.07933	.03588	.0451	.0414	.0432	.0531	.0531	.0531	.0582	.0582	.0582	.03712	.19746	.04301	.04301	.13846	.0604	.0678	.0678	.02371	.02199	.03166	.03437	.09045	.09725	.08895	.08895	.0629	.04332	.05893	.04224	.01724	.03677
	ORM	.04532	.06537	.05837	.05467	.07632	.05277	.06817	.05287	.11832	.05457	.05387	.10687	.12757	.13117	.05027	.1116	.0844	.108	.1374	.1051	.1172	.0993	.1474	.1347	.0462	.1696	.05847	.4202	.3138	.17499	.16519	.19319	.0633	.055	.0695	.04237	.1273	.1083	.0745	.1053	.1276	.0794	.1016	.0723	.1037	.04013
	SUG	.0888														.0865										.0892					.0191	.0191	.0191	.0619	.0475	.0604	.0927	.09	.09	.09	.09	.0543	.04875	.0868	.0934	.066	.0919

Chapter 4
Appendix C

Right Hand Side Values for Equations:
(2), (11) and (12)

	Beg. Inv.	Production (pounds)				Sales (pounds)			
(j)	(B_0)	(Y_1)	(Y_2)	(Y_3)	(Y_4)	(D_1)	(D_2)	(D_3)	(D_4)
JB	1580	3000	3140	4420	3100	3680	3020	3915	2220
JC	2767	2885	2808	4808	962	3923	3269	4692	2346
JD	406	923	462	1846	923	1108	609	1015	1200
JF	4796	22594	25305	27113	17171	22782	27402	27764	19031
JG	2873	8793	21500	15579	13779	11355	21084	14679	11113
JI	0	859	2405	1546	859	859	1340	1821	1031
JJ	1100	1133	667	333	100	1300	800	500	367
JL	936	2308	1923	2256	2410	2359	2808	1987	2436
JN	2833	4322	8077	5783	5091	5795	7756	6013	5192
JO	1312	3272	4879	7002	2948	3523	5660	7282	2697
JP	861	2692	4245	3860	3845	1846	5952	3107	3292
JR	5515	6366	12241	9793	2913	7164	13143	9535	6674
JX	2297	803	332	499	1052	2021	1218	692	803
JY	0	0	3848	740	0	0	3478	962	0
SB	1275	2700	325	1925	1240	1400	1000	325	2600
SD	7583	14699	7731	15807	7560	12876	13418	17037	8248
SE	5958	10562	21936	13393	4211	9134	24251	11744	4924
SF	20205	25433	42184	46215	18700	31149	50646	42050	22820
SH	0	0	0	0	5977	0	0	0	5861
SI	0	0	0	0	18058	0	0	0	17129
SJ	0	0	0	0	33496	0	0	0	33159
SL	0	2331	3848	6031	0	0	4769	7141	0
SM	0	925	518	2572	0	0	666	3349	0
SN	66	1092	0	3410	1616	959	96	3417	627
KB	35856	145944	185542	190152	165256	164898	179862	171315	134280
KC	687	33	1046	948	0	425	1308	687	294
KD	224	0	0	0	0	9	152	9	54
KE	1807	3574	1462	2342	1618	3538	2716	2931	1483
KF	13	753	0	0	766	0	229	537	255
TA	3182	967	0	1591	4430	1591	1310	1591	842
TC	0	2401	10153	7134	0	0	4330	15058	0
TD	0	860	0	2528	0	0	860	2528	0
BA	9425	400	15350	13250	8595	2565	13745	17830	12880
BB	600	5150	2440	0	2480	1570	2935	2460	1195
BC	6015	127725	123200	113465	99135	133740	99440	129755	82567
BD	1300	84892	116075	61170	103807	80064	95892	69450	101700
BF	2094	10688	26457	24208	18157	12782	25410	21591	14938
BG	144	3485	3566	1238	8064	749	5270	432	144
BI	0	4864	1888	2400	0	0	702	8320	0
BJ	0	768	0	1616	0	0	768	1616	0
MA	480	0	2720	2355	2000	40	2920	1920	2480
MB	650	4500	1450	5850	8700	1925	1275	5660	12290
MC	480	0	2418	166	0	425	1809	738	92
MD	583	0	892	0	0	549	652	34	0
ME	9050	10050	6200	9400	14575	14500	9250	10850	14675
MF	1975	0	5175	0	2725	850	3150	1700	640

Chapter 4
Appendix D

Summary of the COMPA, Fully Integrated, Multi-period, Linear Programming Simulation Model

Constraint number and definition	Algebraic expression	△ Rows & vectors
(1) Production, sales and inv.	$-\sum_{t=1}^{T} P_{jt} + \sum_{t=1}^{T} S_{jt} + B_{jT} = B_{j0}$	R = 184 V = 552
(2) Direct labor consumption	$\sum_{j=1}^{N} l_{1j}P_{jT} - RT_{1T} - ET_{1T} = FT_{1T}$	R = 16 V = 32
(3) Direct cost of sugar	$\sum_{j=1}^{N} s_j P_{jT} - GUS_T = 0$	R = 4 V = 4
(4) Direct cost of other raw material	$\sum_{j=1}^{N} r_j P_{jT} - RRM_T = 0$	R = 4 V = 4
Net weight of shipments:		
(5A) Division 1	$\sum_{j=1}^{N-L} S_{jT} - WGJ_T = 0$	R = 4 V = 4
(5B) Division 2	$\sum_{j=N-L+1}^{N} S_{jT} - WGC_T = 0$	R = 4 V = 4
Sales revenue:		
(6A) Division 1	$\sum_{j=1}^{N-L} sp_j S_{jT} - SJ\$_T = 0$	R = 4 V = 4
(6B) Division 2	$\sum_{j=N-L+1}^{N} sp_j S_{jT} - SC\$_T = 0$	R = 4 V = 4
Factoring:		
(7A) Balance	$-x\sum_{t=1}^{T} SJ\$_t - x'\sum_{t=1}^{T} SC\$_t + \sum_{t=1}^{T} FP\$_t - I\sum_{t=1}^{T-1} FB\$_t + (1.-I)FB\$_T = FB\$_0$	R = 4 V = 8

(7B) Upper limit

$$.8\left[(1.-e_T)SJ\$_T + (1.-e_T-e_{T-1})SJ\$_{T-1} + (1.-e_T-e_{T-1}-e_{T-2})SJ\$_{T-2} + (1.-e_T')SC\$_T\right.$$

$$\left. + (1.-e_T'-e_{T-1}')SC\$_{T-1} + (1.-e_T'-e_{T-1}'-e_{T-2}')SC\$_{T-2}\right] - FB\$_T \geq 0$$

R = 4
V = 0

(8) Cash flow

$$R_M \sum_{t=1}^{T-1} RTM_t + w_M R_M(RTM_T) + R_S \sum_{t=1}^{T-1} RTS_t + w_S R_S(RTS_T) + R_C \sum_{t=1}^{T-1} RTC_t + w_C R_C(RTC_T)$$

$$+ R_B \sum_{t=1}^{T-1} RTB_t + w_B R_B(RTB_T) + R_M' \sum_{t=1}^{T-1} ETM_t + w_M R_M'(ETM_T) + R_S' \sum_{t=1}^{T-1} ETS_t + w_S R_S'(ETS_T)$$

$$+ R_C' \sum_{t=1}^{T-1} ETC_t + w_C R_C'(ETC_T) + R_B' \sum_{t=1}^{T-1} ETB_t + w_B R_B\,(ETB_T) + \sum_{t=1}^{T} GUS_t + \sum_{t=1}^{T-1} RRM_t$$

$$+ K \sum_{t=1}^{T-1} WGJ_t + K' \sum_{t=1}^{T-1} WGC_t + m \sum_{t=1}^{T-1} SJ\$_t - x \sum_{t=1}^{T} SJ\$_t - x' \sum_{t=1}^{T} SC\$_t - (y)(e_T)SJ\$_T$$

$$- (y)(e_{T-1})SJ\$_{T-1} - (y)(e_{T-2})SJ\$_{T-2} - (y)(e_{T-3})SJ\$_{T-3} - (y)(e_{T-4})SJ\$_{T-4} - e_T'(SC\$_T)$$

$$- e_{T-1}'(SC\$_{T-1}) - e_{T-2}'(SC\$_{T-2}) - e_{T-3}'(SC\$_{T-3}) - e_{T-4}'(SC\$_{T-4}) + \sum_{t=1}^{T} FP\$_t - LB\$_T$$

$$+ CA\$_T = CA\$_0 - \sum_{t=1}^{T} NMR_t$$

R = 4
V = 4

(9) Maximum overdraft

$$LB\$_T \leq XLC_T$$

R = 4
V = 4

Functionals:

(10A) Contribution basis

$$\text{maximize}\ \Pi = \sum_{t=1}^{M} \sum_{j=1}^{N} \&p_j S_{jt} - .5f_M \sum_{t=1}^{M} ETM_t - .5f_S \sum_{t=1}^{M} ETS_t - .5f_C \sum_{t=1}^{M} ETC_t - .5f_B \sum_{t=1}^{M} ETB_t$$

$$- K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FB\$_t - I' \sum_{t=1}^{M} LB\$_t$$

R = 1
V = 0

(10B) Revenue basis

$$\text{maximize } \Pi = \sum_{t=1}^{M}\sum_{j=1}^{N} nsp_j S_{jt} - f_M \sum_{t=1}^{M} RTM_t - f_S \sum_{t=1}^{M} RTS_t - f_C \sum_{t=1}^{M} RTC_t - f_B \sum_{t=1}^{M} RTB_t - 1.5f_M \sum_{t=1}^{M} ETM_t - 1.5f_S \sum_{t=1}^{M} ETS_t - 1.5f_C \sum_{t=1}^{M} ETC_t - 1.5f_B \sum_{t=1}^{M} ETB_t - \sum_{t=1}^{M} GUS_t - \sum_{t=1}^{M} RRM_t - K \sum_{t=1}^{M} WGJ_t - K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FB\$_t - I' \sum_{t=1}^{M} LB\$_t$$

R = 1
V = 0

Simulation constraints:

(11) Production $P_{jT} = Y_{jT}$

R = 184
V = 0

(12) Sales $S_{jT} = D_{jT}$

R = 184
V = 0

(13) Straight time hours available $PT_{iT} = XT_{iT}$

R = 16
V = 0

Total R = 626
Total V = 624

Chapter 4
Appendix E
COMPA

Fully Integrated Model
Initial Tableau
Financial Constraints

Constraint:		Vector codes: RTM				RTS				RTC				RTB				ETM				ETS				ETC				ETB				GUS				RRM			
#	Code	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
7A.	1XFB																																								
	2XFB																																								
	3XFB																																								
	4XFB																																								
7B.	1XFM																																								
	2XFM																																								
	3XFM																																								
	4XFM																																								
8.	1XCA	1.23				1.23				1.12				.93				1.84				1.84				1.69				1.39				1.							
	2XCA	1.32	1.23			1.32	1.23			1.37	1.12			1.24	.93			1.98	1.84			1.98	1.84			2.06	1.69			1.86	1.39			1.	1.			1.			
	3XCA	1.32	1.32	1.23		1.32	1.32	1.23		1.37	1.37	1.12		1.24	1.24	.93		1.98	1.98	1.84		1.98	1.98	1.84		2.06	2.06	1.69		1.86	1.86	1.39		1.	1.	1.		1.	1.		
	4XCA	1.32	1.32	1.32	1.23	1.32	1.32	1.32	1.23	1.37	1.37	1.37	1.12	1.24	1.24	1.24	.93	1.98	1.98	1.98	1.84	1.98	1.98	1.98	1.84	2.06	2.06	2.06	1.69	1.86	1.86	1.86	1.39	1.	1.	1.	1.	1.	1.	1.	
9.	1XLC																																								
	2XLC																																								
	3XLC																																								
	4XLC																																								

(continued)

Constraint code:	WGJ				WGC				SJ$				SC$				FP$				FB$				LB$				CA$				Sign	Right hand side values
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4		
1XFB									-.775				-.79				1.				.98625												=	175,100
2XFB									-.775	-.775			-.79	-.79			1.	1.			-.01375	.98625											=	175,100
3XFB									-.775	-.775	-.775		-.79	-.79	-.79		1.	1.	1.		-.01375	-.01375	.986										=	175,100
4XFB									-.775	-.775	-.775	-.775	-.79	-.79	-.79	-.79	1.	1.	1.	1.	-.01375	-.01375	-.014	.986									=	175,100
1XFM									.392				.543																				≥	- 89,550
2XFM									.152	.392			.312	.543																			≥	- 39,800
3XFM									.048	.152	.342		.256	.312	.543																		≥	0
4XFM										.048	.152	.392		.256	.312	.543																	≥	0
1XCA									-1.275				-1.11				1.								-1.				1.				=	- 49,793
2XCA	.04				.03				-1.52	-1.275			-1.40	-1.11			1.	1.								-1.				1.			=	- 85,166
3XCA	.04	.04			.03	.03			-1.648	-1.52	-1.275		-1.46	-1.40	-1.11		1.	1.	1.								-1.				1.		=	- 109,931
4XCA	.04	.04	.04		.03	.03	.03		-1.687	-1.648	-1.52	-1.275	-1.67	-1.46	-1.40	-1.11	1.	1.	1.	1.								-1.				1.	=	- 147,909
1XLC																									1.								≤	100,000
2XLC																										1.							≤	100,000
3XLC																											1.						≤	100,000
4XLC																												1.					≤	100,000

Fully Integrated Simulation Problem
Optimal Primal-Dual Solution

578 582 6 1PFT 266213.91 (1) 1 * * * *

	J(H) (2)	BETA(H) (3)	RCW(I) (4)	PI(I) (5)	B(I) (6)
0	C0000	266213.90893651	1PFT	1.00000000	.
0	C0000	285554.62832656	2NSP	.	.
	1PJB	3000.0C0CCC000	1M1L	.95560000	480.00000000
	1PJC	2885.000CC0000	1S0L	.95560000	360.00000000
	1PJD	923.000CC0000	1C1L	.98740000	540.00000000
	1PJF	22594.000CC0C0	1B1L	.09300000	440.00000000
	1PJG	8793.000CC000	2M1L	.07740000	552.00000000
	1PJI	859.000CC000	2S0L	.91600000	414.00000000
	1PJJ	1133.000CC000	2C1L	.94620000	621.00000000
	1PJL	2308.C00CC0C0	2B1L	.95220000	506.00000000
	1PJN	4322.000CC000	3M1L	.87640000	504.00000000
	1PJO	3272.000CC000	3S0L	.87640000	378.00000000
	1PJP	2692.000CC000	3C1L	.90500000	567.00000000
	1PJR	6366.000CC000	3B1L	.81500000	462.00000000
	1PJX	803.000CC000	4M1L	.83680000	480.00000000
	1PJY	.	4S0L	.83680000	360.00000000
	1PSB	27C0.000CC000	4C1L	.86380000	540.00000000
	1PSD	14699.000CC000	4B1L	.77780000	440.00000000
	1PSE	10562.000CC000	1TJB	.	1580.00000000
	1PSF	25433.C00CC000	1TJC	.	2767.00000000
	1PSH	.	1TJD	.	406.00000000
	1PSI	.	1TJF	.	4796.00000000
	1PSJ	.	1TJG	.	2873.00000000
	1PSL	2331.000CC000	1TJI	.22981703	.
	1PSM	925.000CC0C0	1TJJ	.	1100.00000000
	1PSN	1092.000CC0C0	1TJL	.	936.00000000
	1PKB	145944.000CC000	1TJN	.	2833.00000000
	1PKC	33.000CC000	1TJO	.	1312.00000000
	1PKD	.	1TJP	.	861.00000000
	1PKE	3574.000CC0C0	1TJR	.	5515.00000000
	1PKF	753.000CC000	1TJX	.	2297.00000000
	1PTA	967.000CC000	1TJY	.07873738	.
	1PTC	2401.0C0CC000	1TSB	.	1275.00000000
	1PTD	860.000CC000	1TSD	.	7583.00000000
	1PBA	400.000C0000	1TSE	.	5958.00000000
	1PBB	5150.000CC000	1TSF	.	20205.00000000
	1PBC	127725.000CC000	1TSH	.00563135	.
	1PBD	84892.000CC000	1TSI	.00563135	.
	1PBF	10688.000CC000	1TSJ	.00563135	.
	1PBG	3485.000CC000	1TSL	.	.
	1PBI	4864.0C0CC000	1TSM	.	.
	1PBJ	768.000CC000	1TSN	.	66.00000000
	1PMB	4500.000CC000	1TKB	.	35856.00000000

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4BTC	300.000CC000	3DJI	.21530586	1821.00000000
4BBB	2510.000CC000	3DJJ	.21224069	500.00000000
4BBC	24038.000CC000	3DJL	.21520586	1987.00000000
4BBD	20138.000CC000	3DJN	.21665319	6013.00000000
0 CC000	.	3DJO	.	7282.00000000
4BBF	6883.000CC000	3DJP	.19670363	3107.00000000
4BBG	9902.000CC000	3DJR	.22087998	9535.00000000
0 00000	.	3DJX	.	692.00000000
4BBI	130.000CC000	3DJY	.06547930	962.00000000
4BMA	195.000CC000	3DSB	.09647861	325.00000000
4BMD	240.000CC000	3DSD	.11830028	17037.00000000
4BMF	3535.000CC000	3DSE	.13820028	11744.00000000
1RTM	930.000CC000	3DSF	.11780028	42050.00000000
0 00000	.	3DSH	.	.
0 CC000	.	3DSI	.	.
0 00000	.	3DSJ	.	.
2RTM	1382.48699158	3DSL	.03843554	7141.00000000
0 00000	.	3DSM	.	3349.00000000
3RTM	1078.000CC000	3DSN	.05037483	3417.00000000
4RTM	942.000CC000	3DKB	.12239502	171315.00000000
1RTS	1310.000CC000	3DKC	.00955105	687.00000000
2RTS	1906.000CC000	3DKD	.00751529	9.00000000
0 00000	.	3DKE	.	2931.00000000
0 CC000	.	3DKF	.	537.00000000
3RTS	1525.000CC000	3DTA	.12879153	1591.00000000
4RTS	1471.000CC000	3DTC	.05607099	15058.00000000
0 00000	.	3DTD	.	2528.00000000
1RTC	2500.000CC000	3DBA	.00342529	17830.00000000
2RTC	3239.000CC000	3DBB	.11870152	2460.00000000
3RTC	2907.000CC000	3DBC	.10981823	129755.00000000
4RTC	2721.000CC000	3DBD	.12342834	69450.00000000
1RTB	3457.46174291	3DBF	.19907508	21591.00000000
2RTB	5181.000CC000	3DBG	.12182194	432.00000000
3RTB	4984.000CC000	3DBI	.08043644	8320.00000000
0 00000	.	3DBJ	.	1616.00000000
4RTB	3983.000CC000	3DMA	.05013835	1920.00000000
1ETM	94.02984095	3DMB	.00276132	5660.00000000
0 CC000	.	3DMC	.	738.00000000
3ETM	152.24046160	3DMD	.15375559	34.00000000
4ETM	239.55541113	3DME	.00320396	10850.00000000
1ETS	134.34897280	3DMF	.08722815	1700.00000000
2ETS	37.11683325	4DJB	.12565478	2220.00000000
0 00000	.	4DJC	.	2346.00000000
3ETS	237.72085386	4DJD	.20852022	1200.00000000
0 00000	.	4DJF	.	19031.00000000
4ETS	184.69699291	4DJG	.20271713	11113.00000000
1ETC	646.35669474	4DJI	.20905901	1031.00000000
2ETC	926.99238612	4DJJ	.20580591	367.00000000

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	3ETC	774.59134539		4DJL	.20895901	2436.00000000
	4ETC	923.17669519		4DJN	.20934148	5192.00000000
	2ETB	29.46475344		4DJO	.20725901	2697.00000000
	3ETB	131.39690245		4DJP	.19073865	3292.00000000
	4ETB	519.54289248		4DJR	.21369354	6674.00000000
	1GUS	42427.70299724		4DJX	.08690089	803.00000000
	2GUS	55328.19559711		4DJY	.05978053	.
	3GUS	49841.69239672		4DSB	.09226752	2600.00000000
	4GUS	48209.99029756		4DSD	.11275811	8248.00000000
	1RRM	35699.18851445		4DSE	.13265811	4924.00000000
	2RRM	45943.86741533		4DSF	.11225811	22820.00000000
	3RRM	43983.61628480		4DSH	.02484600	5861.00000000
	4RRM	41052.02695451		4DSI	.05714600	17129.00000000
	1WGJ	143301.000CC000		4DSJ	.04504600	33159.00000000
	2WGJ	237901.000CC000		4DSL	.03405218	.
0	CC000	.		4DSM	.	.
	3WGJ	225099.000CC000		4DSN	.04523981	627.00000000
	4WGJ	171872.000CC000		4DKB	.11854092	134280.00000000
0	C0000	.		4DKC	.	294.00000000
0	CC000	.		4DKD	.	54.00000000
	1WGC	400152.000CC000		4DKE	.21282451	1483.00000000
	2WGC	408469.000CC000		4DKF	.12250132	255.00000000
	3WGC	410940.000CC000		4DTA	.12281089	842.00000000
	4WGC	362707.000CC000		4DTC	.05128048	.
0	CC000	.		4DTD	.	.
0	C0000	.		4DBA	.	12880.00000000
	1SJ$	65206.361CC012		4DBB	.11527623	1195.00000000
	2SJ$	107214.053CC017		4DBC	.10628228	82567.00000000
	3SJ$	98969.097CC008		4DBD	.11960191	101700.00000000
	4SJ$	72934.916CC071		4DBF	.19110565	14938.00000000
	1SC$	124618.77599795		4DBG	.11494870	144.00000000
	2SC$	127878.77899792		4DBI	.07523878	.
0	C0000	.		4DBJ	.	.
	3SC$	127550.65499774		4DMA	.04627043	2480.00000000
0	CC000	.		4DMB	.	12290.00000000
	4SC$	112570.41299844		4DMC	.00652873-	92.00000000
	1FP$	143818.08365945		4DMD	.14810380	.
0	CC000	.		4DME	.	14675.00000000
	2FP$	169586.30913249		4DMF	.08389969	640.00000000
	3FP$	181031.62046633	+	1XTM	.85180C00	930.00000000
	4FP$	164853.67452759	+	2XTM	.	1419.00000000
	1FB$	192778.88887909	+	3XTM	.82540000	1078.00000000
	2FB$	200058.51075596	+	4XTM	.81220000	942.00000000
	3FB$	199282.91877535	+	1XTS	.85180C00	1310.00000000
	4FB$	182438.57040239	+	2XTS	.83860C00	1906.00000000
	1LB$	24857.03427823	+	3XTS	.82540000	1525.00000000
	2LB$	24158.21704124	+	4XTS	.81220000	1471.00000000
	3LB$	16943.58762988	+	1XTC	.88280000	2500.00000000

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	4LB$	19701.28471832	+	2XTC	.86900000	3239.00000000
+	1XLC	75142.96572177	+	3XTC	.85520000	2907.00000000
+	2XLC	75841.78295876	+	4XTC	.84140000	2721.00000000
+	3XLC	83056.41237012	+	1XTB	.	4050.00000000
+	4XLC	80298.71528168	+	2XTB	.78400000	5181.00000000
+	2XTM	36.51300842	+	3XTB	.77160000	4984.00000000
+	1XTB	592.53825709	+	4XTB	.75920000	3983.00000000

Footnotes:
(1) Maximum value of the functional.
(2) Coded notation identifying the variables in the basis. Codes preceded by - or - represent slack variables.
(3) The optimum values of the basis variables.
(4) Row ID codes from the initial tableau.
(5) The shadow price of the row named.
(6) The initial right hand side value.

Chapter 4
Appendix G
COMPA

Values of Independent and Dependent Variables used in Regression Analyses to Determine Matrix Coefficients for Aggregate Product Groups
(all numbers are rounded to nearest integer)

Aggregate Model 01:		Ind. Var. # prod.	Process: M1L	S0L	C1L	B1L	SUG	Cost: ORM
Product (JT):	Feb	125787	328	509	818	1719	11008	12156
	July	146085	381	571	955	2349	12570	14775
	Aug	222336	581	864	1451	3932	18901	21464
	Sept	219102	555	834	1432	3808	18503	21295
	Oct	181530	467	694	1197	3089	15636	18411
	Nov	205074	512	792	1354	2980	17790	21252
Product (CG):	Feb	262774	805	861	1904	1417	21055	14473
	July	378661	1123	1234	2732	1549	29857	20924
	Aug	452072	1354	1493	3336	1781	36427	24190
	Sept	395642	1179	1307	2817	1769	31339	21841
	Oct	404789	1194	1321	2987	1854	32574	22055
	Nov	314610	953	1050	2292	1237	24978	16928

		Ind. Var. # sold	Row ID 1PFT	2NSP	XSJ	XSC
Product (JT):	July	143301	28327	61953	65206	-
	Aug	237901	45236	101778	107214	-
	Sept	225099	40129	94035	98969	-
	Oct	171872	28114	69315	72935	-
Product (CG):	July	400152	55595	123019	-	124619
	Aug	408469	57234	126242	-	127879
	Sept	410940	56855	125908	-	127551
	Oct	362707	49729	111123	-	112570

Aggregate Model 02:		Ind. Var. # prod.	Process: M1L	S0L	C1L	B1L	SUG	Cost: ORM
Product (JT):	Feb	63405	-	-	-	628	-	4522
	July	59950	-	-	-	601	-	4134
	Aug	91832	-	-	-	1017	-	6942
	Sept	85578	-	-	-	863	-	5983
	Oct	55153	-	-	-	558	-	3833
	Nov	61636	-	-	-	644	-	4611
Product (ST):	Feb	47641	-	-	-	515	-	4929
	July	57742	-	-	-	637	-	5982
	Aug	76542	-	-	-	807	-	7855
	Sept	89353	-	-	-	1031	-	9487
	Oct	33327	-	-	-	372	-	3519
	Nov	16376	-	-	-	173	-	1708
Product (SJ)[1]								
Product (KB)[2]								
Product (KT):	Feb	2717	-	-	-	13	-	1142
	July	4360	-	-	-	60	-	1744
	Aug	2508	-	-	-	159	-	502
	Sept	3290	-	-	-	149	-	298
	Oct	2384	-	-	-	46	-	335
	Nov	3701	-	-	-	32	-	227
Product (TT):	Feb	953	0	0	-	-	-	167
	July	4228	0	0	-	-	-	732
	Aug	10153	0	0	-	-	-	1677
	Sept	11253	0	0	-	-	-	1945
	Oct	4430	0	0	-	-	-	775
	Nov	2402	0	0	-	-	-	420

Product (BC):	Feb	144170	442	458	1114	374	11481	7665
	July	218167	655	657	1624	568	15956	12782
	Aug	257065	774	785	1965	662	19316	14586
	Sept	187885	560	556	1408	484	13344	11316
	Oct	214017	647	660	1634	553	16310	11969
	Nov	174981	524	526	1322	451	12818	10209
Product (BT):	Feb	11071	-	-	-	528	-	1397
	July	19805	-	-	-	902	-	2183
	Aug	31911	-	-	-	1512	-	3897
	Sept	29462	-	-	-	1364	-	3565
	Oct	26221	-	-	-	1282	-	3189
	Nov	17399	-	-	-	847	-	2128
Product (MC):	Feb	28730	85	57	195	607	1558	2657
	July	14550	16	15	142	273	883	1399
	Aug	15545	24	17	185	252	1103	1313
	Sept	17605	30	19	152	364	1033	1740
	Oct	28000	42	35	277	511	1745	2567
	Nov	12315	34	34	128	169	804	838
Product (MT):	Feb	0	0	0	0	0	0	0
	July	0	0	0	0	0	0	0
	Aug	3310	11	19	23	49	293	310
	Sept	166	.62	1.04	1.1	2.7	14.40	16.9
	Oct	0	0	0	0	0	0	0
	Nov	1735	6.5	10.9	11.5	28.3	151	176

	Ind. Var. # sold	Row ID 1PFT	2NSP	XSJ	XSC
Product (JT)	July 67715	15700	29439	30983	-
	Aug 97539	22469	42682	44918	-
	Sept 83964	19587	36506	38421	-
	Oct 58402	13715	25525	26863	-

Product (ST):	July	55518	8280	21237	22341	-
	Aug	94846	14015	36067	37844	-
	Sept	85063	11676	31997	33665	-
	Oct	39219	5790	14873	15646	-
Product (SJ)[1]						
Product (KB)[2]						
Product (KT):	July	3972	934	3084	3243	-
	Aug	4405	966	3217	3384	-
	Sept	4164	914	3139	3302	-
	Oct	2086	466	1568	1649	-
Product (TT):	July	1591	250	655	689	-
	Aug	6500	662	2328	2449	-
	Sept	19177	1792	6701	7053	-
	Oct	842	133	347	365	-
Product (BC):	July	217939	30158	65940	-	66812
	Aug	212012	29639	64424	-	65272
	Sept	219495	30305	66047	-	66925
	Oct	198342	27872	60534	-	61327
Product (BT):	July	13531	2978	7133	7525	-
	Aug	32510	6648	16424	17403	-
	Sept	31959	6010	15350	16168	-
	Oct	15082	3371	8011	8436	-
Product (MC):	July	17315	1447	4641	-	4709
	Aug	16595	1428	4623	-	4691
	Sept	20130	1627	5382	-	5462
	Oct	30085	2322	7888	-	8005
Product (MT):	July	974	184	404	425	-
	Aug	2461	475	1060	1115	-
	Sept	772	151	342	359	-
	Oct	92	18	41	43	-

1, Coefficients for product group "SJ" could not be determined by regression analysis because this item was produced in only two months and sold in one.
2, Product "KB" is the only item included in group "KB"

Chapter 4
Appendix H
COMPA

Aggregate Product
Regression Equations[1]

j	Coeff.	Variables[2] Ind.	Dep.	Equation term:[3] a	b	r^2
Aggregate Model 01:						
JT	l_M	P	M1L	16.14	.00248	.9925
	l_S	P	SOL	44.15	.003636	.9966
	l_C	P	C1L	- 6.71	.006589	.9993
	l_B	P	B1L	- 758.35	.02039	.9231
	s	P	SUG	627.47	.0824	.9979
	r	P	ORM	311.58	.09772	.9783
	gp	S	1PFT	- 448.26	.18454	.9123
	nsp	S	2NSP	-1235.87	.42667	.9865
	sp	S	XSJ	-1424.71	.4498	.9862
CG	l_M	P	M1L	53.11	.00285	.9978
	l_S	P	SOL	5.09	.003276	.9978
	l_C	P	C1L	- 66.71	.007457	.9942
	l_B	P	B1L	530.51	.00291	.6671
	s	P	SUG	- 431.51	.08097	.9976
	r	P	ORM	506.00	.05315	.9941
	gp	S	1PFT	-6516.53	.155144	.9920
	nsp	S	2NSP	-3889.94	.31717	.9959
	sp	S	XSC	-3930.42	.32127	.9960
Aggregate Model 02:						
JT	l_M	basic coefficient			.0027	-
	l_S	"			.0045	-
	l_C	"			.00625	-
	l_B	P	B1L	- 99.30	.01175	.9753
	s	basic coefficient			.0888	-
	r	P	ORM	- 436.54	.0781	.9750
	gp	S	1PFT	496.25	.22588	.9994
	nsp	S	2NSP	- 172.62	.43834	.9998
	sp	S	XSJ	- 179.253	.46129	.9998
ST	l_M	basic coefficient			.00219	-
	l_S	"			.00356	-
	l_C	"			.00677	-
	l_B	P	B1L	- 15.94	.011311	.9929
	s	basic coefficient			.0865	-
	r	P	ORM	- 32.57	.104914	.9988
	gp	S	1PFT	328.18	.13999	.9870
	nsp	S	2NSP	146.54	.377169	.9996
	sp	S	XSJ	151.14	.39684	.9996

Appendix H (continued

SJ	l_M	basic coefficinet		.00219	-
	l_S	"	"	.00356	-
	l_C	"	"	.00677	-
	l_B	"	"	.0115	-
	s	"	"	.0865	-
	r	weighted average		.117	-
	gp	"	"	.0823	-
	nsp	"	"	.3375	-
	sp	"	"	.355	-
KB	l_M	basic coefficient		.0031	-
	l_S	"	"	.00385	-
	l_C	"	"	.00661	-
	l_B	"	"	.00485	-
	s	"	"	.0892	-
	r	"	"	.0462	-
	gp	"	"	.14548	-
	nsp	"	"	.318	-
	sp	"	"	.322	-
KT	l_M	basic coefficient		.0031	-
	l_S	"	"	.00385	-
	l_C	"	"	.00661	-
	l_B	weighted average[4]		.0242	-
	s	basic coefficient		.0892	-
	r	weighted average[4]		.224	-
	gp	S	1PFT 10.68	.2213	.9839
	nsp	S	2NSP 40.98	.74137	.9903
	sp	S	XSJ 42.42	.77995	.9904
TT	l_M	basic coefficient		0	-
	l_S	"	"	0	-
	l_C	"	"	.00614	-
	l_B	"	"	.035233	-
	s	"	"	.0191	-
	r	P	ORM 15.03	.16834	.9985
	gp	S	1PFT 82.40	.0892	.9993
	nsp	S	2NSP 81.29	.3453	.9999
	sp	S	XSJ 84.94	.36344	.9999
BC	l_M	P	M1L 7.78	.002972	.9985
	l_S	P	SOL 15.07	.002969	.9890
	l_C	P	C1L 8.05	.007539	.9954
	l_B	P	B1L 2.37	.002573	.9995
	s	P	SUG 505.08	.072052	.9702
	r	P	ORM - 388.06	.05923	.9694
	gp	S	1PFT 4959.84	.11575	.9931
	nsp	S	2NSP 7617.44	.26714	.9957
	sp	S	XSC 7612.40	.27116	.9958

BT	l_M	basic coefficient			.00398	-
	l_S	"	"		.00396	-
	l_C	"	"		.00679	-
	l_B	P	B1L	8.99	.046965	.9937
	s	basic coefficient			.09	-
	r	P	ORM	- 28.83	.1217	.9895
	gp	S	1PFT	620.79	.1782	.9815
	nsp	S	2NSP	853.86	.46917	.9929
	sp	S	XSJ	866.84	.4968	.9916
MC	l_M	P	M1L	- 13.54	.002675	.6061
	l_S	P	SOL	- 2.01	.00162	.5165
	l_C	P	C1L	59.92	.006163	.658
	l_B	P	B1L	- 82.73	.02289	.9495
	s	P	SUG	183.34	.05162	.9369
	r	P	ORM	- 199.19	.1003	.9647
	gp	S	1PFT	287.18	.06747	.9983
	nsp	S	2NSP	422.50	.2478	.9981
	sp	S	XSC	427.46	.2515	.9981
MT	l_M	P	M1L	.31	.0033	.994
	l_S	P	SOL	.39	.005712	.9967
	l_C	P	C1L	- .23	.006966	.9992
	l_B	P	B1L	1.09	.014726	.9961
	s	P	SUG	- 1.12	.088614	.9999
	r	P	ORM	5.71	.09322	.9975
	gp	S	1PFT	- .28	.192864	.9998
	nsp	S	2NSP	- .21	.42983	.9994
	sp	S	XSJ	- .55	.45225	.9994

1, All regression equations and coefficients of determination were calculated using the Harvard Business School time-share computer and program LINREG***.

2, Independent variables are identified as either production (P) or sales (S) volume. Dependent variables are identified by row ID codes from Appendix A.

3, All a and b values are rounded to the nearest significant digit.

4, Data irregularities prevented determination of these coefficients by regression analysis.

Chapter 4
Appendix I

Revised Right Hand Side Values

Constraint: #	Row ID	Range of T	Value	Constraint #	Row ID	Range of T	Value
Aggregate Model 01:							
10C	GPF	1,4	6965	3	SUG	1,4	- 195.5
10D	SPN	1,4	5126	4	ORM	1,4	- 817.6
2	M1L	1	411	6A	XSJ	1,4	1425
	"	2	483	6B	XSC	1,4	3930
	"	3	435	11	YJT	1	146085
	"	4	411.		"	2	222336
	SOL	1	311		"	3	219102
	"	2	365		"	4	181530
	"	3	329		YCG	1	378661
	"	4	311		"	2	452072
	C1L	1	613.7		"	3	395642
	"	2	694.7		"	4	404789
	"	3	640.7	12	DJT	1	143301
	"	4	613.7		"	2	237901
	B1L	1	668		"	3	225099
	"	2	734		"	4	171872
	"	3	690		DCG	1	400152
	"	4	668		"	2	408469
1	TJT	1,4	70577		"	3	410940
	TCG	1,4	65351		"	4	362707
Aggregate Model 02:							
10C	GPF	1,4	- 6785.6	3	SUG	1,4	- 687
10D	SPN	1,4	- 8990	4	ORM	1,4	1064
1	TJT	1,4	27276	6A	XSJ	1,4	- 966
	TST	1,4	35087	6B	XSC	1,4	- 8040
	TSJ	1,4	0	2	M1L	1	485.7
	TKB	1,4	35856		"	2	557.7
	TKT	1,4	2731		"	3	509.7
	TTT	1,4	3182		"	4	485.7
	TBC	1,4	17340		SOL	1	346.9
	TBT	1,4	1238		"	2	400.9
	TMC	1,4	12155		"	3	364.9
	TMT	1,4	1063		"	4	346.9
					C1L	1	471.9
					"	2	552.9
					"	3	498.9
					"	4	471.9
					B1L	1	625.5
					"	2	691.5
					"	3	647.5
					"	4	625.5

Appendix I (continued)

Constraint: #	Row ID	Values, Period T = 1	2	3	4
11	YJT	59950	91832	85578	55153
	YST	57742	76542	89353	33327
	YSJ	0	0	0	57531
	YKB	145944	185542	190152	165256
	YKT	4360	2508	3290	2384
	YTT	4228	10153	11253	4430
	YBC	218167	257065	187885	214017
	YBT	19805	31911	29462	26221
	YMC	14550	15545	17605	28000
	YMT	0	3310	166	0
12	DJT	67715	97539	83964	58402
	DST	55518	94846	85063	39219
	DSJ	0	0	0	56149
	DKB	164898	179862	171315	134280
	DKT	3972	4405	4164	2086
	DTT	1591	6500	19177	842
	DBC	217939	212012	219495	198342
	DBT	13531	32150	31959	15082
	DMC	17315	16595	20130	30085
	DMT	974	2461	772	92

Chapter 4
Appendix J

COMPA: Aggregate Model 01
Fully Integrated Simulation Problem
Optimal Primal-Dual Solution

1PFT 266343.23

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0 0CCC0	266343.23186241	1PFT	1.CCCCCCCC	.
0 0CCC0	287666.28124644	2NSP	.	.
0 0CCC0	.	1DLC	.	.
0 0CCC0	.	2DLC	.	.
0 0CCC0	.	3DLC	.	.
0 0CCC0	.	4DLC	.	.
1PJT	146085.C0C0CCC0	1GPF	1.CCCCCCCC-	6965.C0CCC0C0
1PCG	378661.C0CCCCC0	2GPF	1.CCCCCCCC-	6965.C0CCCCC0
2PJT	222336.C0CCCCC0	3GPF	1.CCCCCCCC-	6965.C0CCC0C0
2PCG	452C72.CCCCCC00	4GPF	1.CCCCCCCC-	6965.C0CCC0C0
3PJT	219102.CCCCCCCC	1SPN	.	5126.C0CCC0C0
3PCG	395642.C0CCCCC0	2SPN	.	5126.C0CCC0C0
4PJT	181530.C0CCCCC0	3SPN	.	5126.C0CCC0C0
4PCG	4C4789.C0CCCCC0	4SPN	.	5126.C0CCC0C0
1SJT	143301.C0CCCCC0	1M1L	.9556CCCC	411.C0CCCCCC
1SCG	4C0152.C0CCCCC0	1SCL	.9556CCCC	311.C0CCCCC0
2SJT	237901.CCC00C00	1C1L	.9874CCCC	613.70CC00C0
2SCG	408469.C0CCCCC0	1B1L	.C930CCC0	668.00CCC0C0
3SJT	225099.C0CCCCC0	2M1L	.C774CCCC	483.C0CCCCC0
3SCG	41C94C.C0CCCCC0	2SCL	.916CCCCC	365.CCCCCCC0
4SJT	171872.C0CCCC00	2C1L	.9462CCCC	694.70CCC0C0
4SCG	362707.C0CCCC00	2B1L	.C682CCCC	734.C0CCCCCC
1BJT	73361.C0CCCCC0	3M1L	.8764CCCC	435.C0CCCCC0
1BCG	43860.C0CCCCC0	3SCL	.8764CCCC	329.C0CCCCC0
2BJT	57796.C0CC0C00	3C1L	.9C5CCCCC	640.70CCCCC0
2BCG	87463.C0CC0C00	3B1L	.0434CCC0	690.C0CCCCC0
3BJT	51799.C0CCCCC0	4M1L	.8368CCCC	411.C0CCCCC0
3BCG	72165.C0CCCCC0	4SCL	.8368CCCC	311.C0CCC0C0
4BJT	61457.C0CCCC00	4C1L	.8638CCCC	613.70CC00C0
4BCG	114247.C0CC0CC0	4B1L	.77780CCC	668.C0CCC0C0
1RTM	930.C0CCCCC0	1TJT	.	70577.C0CCC0C0
2RTM	1356.79847442	1TCG	.	65351.C0CCC0C0
3RTM	1C78.C0CC0C00	2TJT	.	70577.00CC00C0
4RTM	942.C0C0CC00	2TCG	.	65351.C0CCC0C0
1RTS	131C.C0C00C00	3TJT	.	70577.C0CCCCC0
2RTS	190E.C0CCCCC0	3TCG	.	65351.C0CCCCC0
3RTS	1525.C0C00C00	4TJT	.	70577.00CC00C0
4RTS	1471.C0C0CC00	4TCG	.	65351.C0CCC0C0
1RTC	25CC.C0CCCCC0	1SUG	.C8CCCCCC	195.50CCCCC0-
2RTC	3239.C0CCCCC0	2SUG	.C6C0CCCC	195.50CC00CC-
3RTC	29C7.C0CCCCC0	3SUG	.C4C0CCC0	195.50CC00C0-
4RTC	2721.C0CCCCC0	4SUG	.C2C0CCCC	195.50CCC0C0-
1RTB	3412.57666247	1ORM	.C6CCCCCC	817.60CCC0CC-

2RTB	5114.96C56250		2ORM	.C4CCCCCC	817.60CCCOCO-
3RTB	4928.808C0196		3ORM	.C2CCCCCO	817.60CCCOCO-
4RTB	3983.COCCCCOO		4ORM	.	817.60CCOOCO-
1ETM	100.47464551		1XWJ	.0424CCCC	.
3ETM	157.95265501		2XWJ	.0416CCCC	.
4ETM	250.84304508		3XWJ	.C408CCCC	.
1ETS	150.65849842		4XWJ	.C4OOCCCC	.
2ETS	18.40157130		1XWC	.0318CCCO	.
3ETS	238.77806710		2XWC	.C312CCCC	.
4ETS	204.13184681		3XWC	.C3C6CCCC	.
1ETC	672.52914143		4XWC	.C3CCCCCC	.
2ETC	9C2.37280718		1XSJ	.C637O452-	1425.COCCCCCO
3ETC	846.26547120		2XSJ	.C45588C4-	1425.COCCCOCO
4ETC	879.91274232		3XSJ	.C279C192-	1425.COCCCOCO
4ETB	228.33269240		4XSJ	.C1224224-	1425.OOCCCOCO
1GLS	42893.08517581		1XSC	.C5548169-	393C.COCCCCCO
2GLS	55120.25624760		2XSC	.C381C737-	393C.COCCCOCO
3GLS	5C284.63754699		3XSC	.C2333856-	393C.COCCOOCO
4GLS	47929.33733656		4XSC	.CC95C596-	3930.OOCCCOCO
1RRM	35218.858,34578		1XFB	.C2CCCCCC-	1751CC.COCCCOCO
2RRM	46571.90071451		2XFB	.C2CCCCCC-	1751CC.COCCCOCO
3RRM	43256.61973497		3XFB	.C2CCCCCC-	1751CC.COCCCOCO
4RRM	40C71.24694525		4XFB	.C20CCCCC-	1751CO.COCCCOCO
1WGJ	143301.COCOCCCO	-	1XFM	.CC515CCC-	895SC.COCCCOCO-
2WGJ	2379C1.CCCCCCOO	-	2XFM	.CC5425CC-	398CO.COCCCOCO-
3WGJ	225099.COCCCCCO	-	3XFM	.CC544CCC-	.
4WGJ	171872.COCOOCOO	-	4XFM	.CO572OCC-	.
1WGC	4CC152.COCOCCCO	+	1XLC	.	1CCCCC.COCCCOCO
2WGC	408469.COCCCCCO	+	2XLC	.	1CCCCC.COCCCOCO
3WGC	410940.COCCCCCO	+	3XLC	.	1CCCCC.COCCCOCO
4WGC	362707.COOOOCOO	+	4XLC	.	1CCCCC.COCCOOCO
1SJ$	63031.78980C31		1XCA	.C2COCCCC	49793.COCCCOCO-
2SJ$	1C5582.86980C51		2XCA	.02CCCCCC	85166.COCCCOCO-
3SJ$	99824.53C2CC48		3XCA	.02COCCCC	1C9931.COCCCOCO-
4SJ$	75883.C256CC37		4XCA	.C2CCCOCC	147CC9.COCCCOCO-
1SC$	124626.83303432		1YJT	.C267C19C-	146C85.COCCCCCO
2SC$	127298.83562421		1YCG	.02315428-	378661.COCCCOCO
3SC$	128C92.69379417		2YJT	.C2CCCC44-	222736.COCCCOCO
4SC$	112596.87788486		2YCG	.01745988-	452072.COOCCOCO
1FP$	142975.55219497		3YJT	.01745843-	219102.COCCCOCO
2FP$	168280.65167238		3YCG	.C1654551-	395642.COCCCOCO
3FP$	180743.35827725		4YJT	.C2831679-	18153C.COCCCOCO
4FP$	166032.80928999		4YCG	.01545C39-	4C4789.COCCCCCO
1FB$	181930.83193953		1DJT	.17C79429	143301.COCCCCCO
2FB$	198776.15666275		1DCG	.14116860	4CC152.COCCCOCO
3FB$	199381.37666111		2DJT	.1634455C	2379C1.COCCOOCO
4FB$	183680.97945244		2DCG	.13618676	4C8469.COCCCOCO
1LB$	27287.87623934		3DJT	.15629C28	225099.COCCCOCO

	2LB$	27638.00345772		3DCG	.13204198	41C94C.COCCCOCO
	3LB$	2C210.68778695		4DJT	.15C04656	171872.COCCCOCO
	4LB$	18838.81852870		4DCG	.12819798	3627C7.COCCCOCO
	1GP$	81560.94842195	+	1XTM	.8518CCCO	930.COCCCOCO
	2GP$	100308.76506868	+	2XTM	.	1419.COCCCOCO
	3GP$	98329.64481281	+	3XTM	.8254CCCC	1078.COCCCOCO
	4GP$	81C24.07368199	+	4XTM	.8122CCCC	942.COCCOOCO
	1SP$	182932.44750594	+	1XTS	.8518CCCC	131C.COCCCOCO
	2SP$	225933.33239459	+	2XTS	.8386CCCC	1906.COCCCOCO
	3SP$	221254.83012476	+	3XTS	.8254CCCC	1525.COCCCOCO
	4SP$	183246.40542574	+	4XTS	.8122CCCC	1471.00CCCOCO
+	1XLC	72712.12376C66	+	1XTC	.8828CCCC	25C0.COCCCOCO
+	2XLC	72361.99654228	+	2XTC	.869CCCCC	3239.COCCCOCO
+	3XLC	79789.31221305	+	3XTC	.8552CCCC	29C7.COCCCOCO
+	4XLC	81161.18147130	+	4XTC	.8414CCCO	2721.00CCOOCO
+	2XTM	62.20152558	+	1XTB	.	405C.COCCCCCO
+	1XTB	637.42333753	+	2XTB	.	5181.COCCCOCO
+	2XTB	66.03943750	+	3XTB	.	4984.COCCOOCO
+	3XTB	55.19199804	+	4XTB	.7592CCCC	3983.COCCOOCO

Chapter 4
Appendix K

COMPA: Aggregate Model 02
Fully Integrated Simulation Problem
Optimal Primal-Dual Solution

1PFT 266284.99

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0 00000	266284.98980510	1PFT	1.00000000	.
0 00000	287407.21764410	2NSP	.	.
0 00000	.	1DLC	.	.
0 00000	.	2DLC	.	.
0 00000	.	3DLC	.	.
0 00000	.	4DLC	.	.
1PJT	59950.00000000	1GPF	1.00000000-	6785.60000000-
1PST	57742.00000000	2GPF	1.00000000-	6785.60000000-
1PSJ	.	3GPF	1.00000000-	6785.60000000-
1PKB	145944.00000000	4GPF	1.00000000-	6785.60000000-
1PKT	4360.00000000	1SPN	.	8990.00000000-
1PTT	4228.00000000	2SPN	.	8990.00000000-
1PBC	218167.00000000	3SPN	.	8990.00000000-
1PBT	19805.00000000	4SPN	.	8990.00000000-
1PMC	14550.00000000	1M1L	.95560000	485.70000000
2PJT	91832.00000000	1S0L	.95560000	346.90000000
2PST	76542.00000000	1C1L	.98740000	471.90000000
2PSJ	.	1B1L	.09300000	625.50000000
2PKB	185542.00000000	2M1L	.07740000	557.70000000
2PKT	2508.00000000	2S0L	.91600000	400.90000000
2PTT	10153.00000000	2C1L	.94620000	552.90000000
2PBC	257065.00000000	2B1L	.06820000	691.50000000
2PBT	31911.00000000	3M1L	.87640000	509.70000000
2PMC	15545.00000000	3S0L	.87640000	364.90000000
2PMT	3310.00000000	3C1L	.90500000	498.90000000
3PJT	85578.00000000	3B1L	.81500000	647.50000000
3PST	89353.00000000	4M1L	.83680000	485.70000000
3PSJ	.	4S0L	.83680000	346.90000000
3PKB	190152.00000000	4C1L	.86380000	471.90000000
3PKT	3290.00000000	4B1L	.77780000	625.50000000
3PTT	11253.00000000	1TJT	.	27276.00000000
3PBC	187885.00000000	1TST	.	35087.00000000
3PBT	29462.00000000	1TSJ	.00563135	.
3PMC	17605.00000000	1TKB	.	35856.00000000
3PMT	166.00000000	1TKT	.	2731.00000000
4PJT	55153.00000000	1TTT	.	3182.00000000
4PST	33327.00000000	1TBC	.	17340.00000000
4PSJ	57531.00000000	1TBT	.	1238.00000000
4PKB	165256.00000000	1TMC	.	12155.00000000
4PKT	2384.00000000	1TMT	.	1063.00000000
4PTT	4430.00000000	2TJT	.	27276.00000000
4PBC	214017.00000000	2TST	.	35087.00000000
4PBT	26221.00000000	2TSJ	.00547857	

4PMC	28000.00000000	2TKB	.	35856.00000000
1SJT	67715.00000000	2TKT	.	2731.00000000
1SST	55518.00000000	2TTT	.	3182.00000000
1SSJ	.	2TBC	.	17340.00000000
1SKB	164898.00000000	2TBT	.	1238.00000000
1SKT	3972.00000000	2TMC	.	12155.00000000
1STT	1591.00000000	2TMT	.	1063.00000000
1SBC	217939.00000000	3TJT	.	27276.00000000
1SBT	13531.00000000	3TST	.	35087.00000000
1SMC	17315.00000000	3TSJ	.05140518	.
1SMT	974.00000000	3TKB	.	35856.00000000
2SJT	97539.00000000	3TKT	.	2731.00000000
2SST	94846.00000000	3TTT	.	3182.00000000
2SSJ	.	3TBC	.	17340.00000000
2SKB	179862.00000000	3TBT	.	1238.00000000
2SKT	4405.00000000	3TMC	.	12155.00000000
2STT	6500.00000000	3TMT	.	1063.00000000
2SBC	212012.00000000	4TJT	.	27276.00000000
2SBT	32150.00000000	4TST	.	35087.00000000
2SMC	16595.00000000	4TSJ	.	.
2SMT	2461.00000000	4TKB	.	35856.00000000
3SJT	83964.00000000	4TKT	.	2731.00000000
3SST	85063.00000000	4TTT	.	3182.00000000
3SSJ	.	4TBC	.	17340.00000000
3SKB	171315.00000000	4TBT	.	1238.00000000
3SKT	4164.00000000	4TMC	.	12155.00000000
3STT	19177.00000000	4TMT	.	1063.00000000
3SBC	219495.00000000	1SUG	.08000000	687.00000000-
3SBT	31959.00000000	2SUG	.06000000	687.00000000-
3SMC	20130.00000000	3SUG	.04000000	687.00000000-
3SMT	772.00000000	4SUG	.02000000	687.00000000-
4SJT	58402.00000000	1ORM	.06000000	1064.00000000
4SST	39219.00000000	2ORM	.04000000	1064.00000000
4SSJ	56149.00000000	3ORM	.02000000	1064.00000000
4SKB	134280.00000000	4ORM	.	1064.00000000
4SKT	2086.00000000	1XWJ	.04240000	.
4STT	842.00000000	2XWJ	.04160000	.
4SBC	198342.00000000	3XWJ	.04080000	.
4SBT	15082.00000000	4XWJ	.04000000	.
4SMC	30085.00000000	1XWC	.03180000	.
4SMT	92.00000000	2XWC	.03120000	.
1BJT	19511.00000000	3XWC	.03060000	.
1BST	37311.00000000	4XWC	.03000000	.
1BKB	16902.00000000	1XSJ	.06370452-	966.00000000-
1BKT	3119.00000000	2XSJ	.04558804-	966.00000000-
1BTT	5819.00000000	3XSJ	.02790192-	966.00000000-
1BBC	17568.00000000	4XSJ	.01224224-	966.00000000-
1BBT	7512.00000000	1XSC	.05548169-	8040.00000000-

1BMC	9390.00000000		2XSC	.03810737-	8040.00000000-
1BMT	89.00000000		3XSC	.02333856-	8040.00000000-
2BJT	13804.00000000		4XSC	.00950596-	8040.00000000-
2BST	19007.00000000		1XFB	.02000000-	175100.00000000
2BKB	22582.00000000		2XFB	.02000000-	175100.00000000
2BKT	1222.00000000		3XFB	.02000000-	175100.00000000
2BTT	9472.00000000		4XFB	.02000000-	175100.00000000
2BBC	62621.00000000	-	1XFM	.00515000-	89550.00000000-
2BBT	7273.00000000	-	2XFM	.00542500-	39800.00000000-
2BMC	8340.00000000	-	3XFM	.00544000-	.
2BMT	938.00000000	-	4XFM	.00572000-	.
3BJT	15418.00000000	+	1XLC	.	100000.00000000
3BST	23297.00000000	+	2XLC	.	100000.00000000
3BKB	41419.00000000	+	3XLC	.	100000.00000000
3BKT	348.00000000	+	4XLC	.	100000.00000000
3BTT	1548.00000000		1XCA	.02000000	49793.00000000-
3BBC	31011.00000000		2XCA	.02000000	85166.00000000-
3BBT	4776.00000000		3XCA	.02000000	109931.00000000-
3BMC	5815.00000000		4XCA	.02000000	147909.00000000-
3BMT	332.00000000		1YJT	.02593432-	59950.00000000
4BJT	12169.00000000		1YST	.02644616-	57742.00000000
4BST	17405.00000000		1YSJ	.03532621	.
4BSJ	1382.00000000		1YKB	.02352718-	145944.00000000
4BKB	72395.00000000		1YKT	.03599473-	4360.00000000
4BKT	646.00000000		1YTT	.02096771-	4228.00000000
4BTT	5136.00000000		1YBC	.02267848-	218167.00000000
4BBC	46686.00000000		1YBT	.03316166-	19805.00000000
4BBT	15915.00000000		1YMC	.02246602-	14550.00000000
0.00000	.		1YMT	.	.
4BMC	3730.00000000		2YJT	.01949808-	91832.00000000
4BMT	240.00000000		2YST	.01999421-	76542.00000000
1RTM	930.00000000		2YSJ	.03639321	.
2RTM	1384.33721339		2YKB	.01755169-	185542.00000000
3RTM	1078.00000000		2YKT	.02598336-	2508.00000000
4RTM	942.00000000		2YTT	.01609216-	10153.00000000
1RTS	1310.00000000		2YBC	.01695084-	257065.00000000
2RTS	1906.00000000		2YBT	.02383112-	31911.00000000
3RTS	1525.00000000		2YMC	.01619269-	15545.00000000
4RTS	1471.00000000		2YMT	.02212879-	3310.00000000
1RTC	2500.00000000		3YJT	.02665658-	85578.00000000
2RTC	3239.00000000		3YST	.02594290-	89353.00000000
3RTC	2907.00000000		3YSJ	.02506653	.
4RTC	2721.00000000		3YKB	.02051778-	190152.00000000
1RTB	3518.87280333		3YKT	.03984403-	3290.00000000
2RTB	5136.28198257		3YTT	.03840240-	11253.00000000
3RTB	4984.00000000		3YBC	.01819316-	187885.00000000
4RTB	3983.00000000		3YBT	.05741404-	29462.00000000
1ETM	104.69985231		3YMC	.03206780-	17605.00000000

	3ETM	162.00802411	3YMT	.03161300-	166.00000000
	4ETM	255.19422231	4YJT	.02233886-	55153.00000000
	1ETS	146.84354414	4YST	.02118722-	33327.00000000
	2ETS	36.50918681	4YSJ	.02133423-	57531.00000000
	3ETS	262.01775932	4YKB	.01708181-	165256.00000000
	4ETS	183.76861460	4YKT	.03213224-	2384.00000000
	1ETC	682.07881879	4YTT	.03308996-	4430.00000000
	2ETC	879.13928725	4YBC	.01492594-	214017.00000000
	3ETC	807.79532252	4YBT	.05083877-	26221.00000000
	4ETC	866.29426941	4YMC	.02775390-	28000.00000000
0	00000	.	4YMT	.	.
	3ETB	55.57933434	1DJT	.21286626	67715.00000000
	4ETB	516.43754341	1DST	.12287050	55518.00000000
0	00000	.	1DSJ	.	.
	1GUS	42746.00428311	1DKB	.13154510	164898.00000000
	2GUS	54920.32951881	1DKT	.22858634	3972.00000000
	3GUS	50597.86964262	1DTT	.06995277	1591.00000000
	4GUS	47707.50728321	1DBC	.09899442	217939.00000000
	1RRM	34898.69842281	1DBT	.16744841	13531.00000000
	2RRM	45958.64536407	1DMC	.04961365	17315.00000000
	3RRM	42905.54483703	1DMT	.17927437	974.00000000
	4RRM	41061.35719275	2DJT	.20530931	97539.00000000
	1WGJ	143301.00000000	2DST	.11648116	94846.00000000
0	00000	.	2DSJ	.	.
	2WGJ	237901.00000000	2DKB	.12655057	179862.00000000
	3WGJ	225099.00000000	2DKT	.21525639	4405.00000000
	4WGJ	171872.00000000	2DTT	.06416852	6500.00000000
	1WGC	400152.00000000	2DBC	.09488320	212012.00000000
	2WGC	408469.00000000	2DBT	.15924814	32150.00000000
	3WGC	410940.00000000	2DMC	.04584400	16595.00000000
	4WGC	362707.00000000	2DMT	.17188119	2461.00000000
	1SJ$	65072.90221039	3DJT	.19795088	83964.00000000
	2SJ$	106481.59895045	3DST	.11026260	85063.00000000
0	00000	.	3DSJ	.	.
	3SJ$	99897.92336018	3DKB	.12239502	171315.00000000
	4SJ$	72870.15832090	3DKT	.20226210	4164.00000000
	1SC$	124588.21773869	3DTT	.05854067	19177.00000000
	2SC$	127618.38041844	3DBC	.09147848	219495.00000000
	3SC$	127784.38919864	3DBT	.15126167	31959.00000000
	4SC$	112626.95421922	3DMC	.04272965	20130.00000000
	1FP$	143758.47355447	3DMT	.16468264	772.00000000
	2FP$	169196.06956286	4DJT	.19152722	58402.00000000
	3FP$	181197.54024757	4DST	.10484821	39219.00000000
	4FP$	165016.40891875	4DSJ	.04664600	56149.00000000
	1FB$	182709.97989874	4DKB	.11854092	134280.00000000
	2FB$	199600.17242689	4DKT	.19084834	2086.00000000
	3FB$	199567.13007095	4DTT	.05364932	842.00000000
	4FB$	182556.17412147	4DBC	.08832764	198342.00000000

	1LB$	25479.91059181		4DBT	.14428194	15082.00000000
	2LB$	24320.21810952		4DMC	.03985075	30085.00000000
	3LB$	16840.71985890		4DMT	.15840055	92.00000000
	4LB$	17934.09664872	+	1XTM	.85180000	930.00000000
	1GP$	83856.89374689	+	2XTM	.	1419.00000000
	2GP$	101679.80712509	+	3XTM	.82540000	1078.00000000
	3GP$	97822.84634868	+	4XTM	.81220000	942.00000000
	4GP$	77853.54335907	+	1XTS	.85180000	1310.00000000
	1SP$	184821.39473473	+	2XTS	.83860000	1906.00000000
	2SP$	227115.27089688	+	3XTS	.82540000	1525.00000000
	3SP$	221015.10027978	+	4XTS	.81220000	1471.00000000
	4SP$	180426.40279319	+	1XTC	.88280000	2500.00000000
+	1XLC	74520.08940819	+	2XTC	.86900000	3239.00000000
+	2XLC	75679.78189048	+	3XTC	.85520000	2907.00000000
+	3XLC	83159.28014110	+	4XTC	.84140000	2721.00000000
+	4XLC	82065.90335128	+	1XTB	.	4050.00000000
+	2XTM	34.66278661	+	2XTB	.	5181.00000000
+	1XTB	531.12719667	+	3XTB	.77160000	4984.00000000
+	2XTB	44.71801743	+	4XTB	.75920000	3983.00000000

Chapter 5

COMPA: THE PLANNING MODELS

The COMPA fully integrated, multi-period, linear programming models have been tested and found to be adequately representative of the firm. Validation of the comprehensive model successfully completed two of the three tests specified in Chapter 2. First, company records and experience provided the information necessary to permit formulation of a fully integrated linear programming model of the firm. Second, when forced to simulate actual operations during a recent period, this model predicted levels of profit, cash flow and demand for external funds that approximated actual experience. It has also been shown that Aggregate Model 02 can be used as a proxy for the comprehensive model. Therefore, usability of the COMPA fully integrated model has been established. This chapter will test and evaluate its usefulness as a tool for short term financial planning.

Aggregate Model 02 will be the basic vehicle for all remaining tests. When minor modifications are incorporated to reflect various planning situations it will be redesignated to identify the specific formulation used for each test. In addition, equation 10A-1, (1PFT) will be the operational objective function in all tests. Although other functionals will usually be included, they will be idling and may not convey meaningful information. 1PFT is used because it permits the model to optimize without the possibly distorting effects of gross manufacturing costs that are included in 10B-1 (2NSP) and it provides a value that can be compared directly from problem to problem without numerous corrections.

The purpose of the tests in this chapter will be twofold. First, "what if" type planning problems will be specified and the model modified as necessary to solve them. The solutions to these problems will be compared to evaluate the appropriateness and realism of suggested operating plans and financial

estimates. Second, optimal solutions to the "what if" problems will be subjected to parametric analysis to ascertain the impact of changes in levels of demand and availability of external funds. Parametric variations will also be used to determine if the productive opportunity locus of funds can be defined.

5.1 The Dual Evaluators

Before starting the planning tests, a discussion of the composition and interpretation of shadow prices or dual evaluators appears to be warranted. Shadow prices are the variables in the optimal solution to the "dual" problem.[1] In the primal problem the object is to determine profit maximizing output levels for the various products subject to a number of input limitations. Using the same costs, prices, factor consumption relationships and resource limitations, the dual problem imputes all profits to the scarce resources. To quote from Baumol,[2] "The dual problem requires us to find the very smallest valuation of the company's stock of inputs which completely accounts for all the profits of each of the outputs." These imputed costs are true marginal values and they provide a means of determining increments (or decrements) in profits that will occur if additional resources are made available and optimally employed. The dual problem clearly shows that marginal analysis is always implicitly involved in the search for an optimal solution to a linear programming problem.

Shadow prices for Aggregate Model 02 could be obtained directly by solving the problem restated as its dual. However, this extra step is unnecessary. The standard LP-90 output format shows optimal solutions to both the primal and dual problems. In the dual problem the shadow price of each stipulation reflects the net result of all cost/profit interactions that occur when one additional unit of resource is made available. Because their derivation in a large model is usually obscure, several shadow prices will be dissected in order that explicit interrelationships can be defined and interpreted.

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[1] For a thorough and detailed description of duality and the formulation of dual problems, see Baumol, *op. cit.*, Chapter 6.

[2] *Ibid.*, Page 109

The shadow price analysis will be based on Optimum Model 02, a slightly modified version of Aggregate Model 02.[3] Two basic changes have been incorporated in this model to facilitate the discussion. First, equation (12) has been changed to a Type-I inequality and appears as follows:

$$S_{jT} \leqslant D_{jT} \qquad \begin{matrix} T = 1,M \\ j = 1,N \end{matrix} \tag{12-1}$$

In this form the constraint becomes an upper limit on demand and actual sales (from Chapter 4, Appendix I) are used as the right-hand side values D_{jT}.

The second change eliminates the mandatory production schedule, equation (11). This deletion permits the model to develop an optimal production schedule. However, to insure realism, a new constraint is added to require that ending inventory balances ($4B_j$) equal ending inventory from Aggregate Model 02. This constraint has the form:

$$B_{jT} = F_{jT} \qquad \begin{matrix} T = M \\ j = 1,N \end{matrix} \tag{11-1}$$

Where:

F_{jT} = The ending inventory balance $4B_j$ from Aggregate Model 02 (Chapter 4, Appendix K).

In Aggregate Model 02, beginning inventory, production and sales were set and ending inventory was determined. Beginning inventory, sales, and ending inventory are specified in Optimum Model 02. Because inventories are the same, if all possible sales are accepted total production must also be identical in both models. For this reason, Optimum Model 02 can optimize only to the extent that production can be profitably rescheduled within the planning cycle. The optimal primal-dual solution to Optimum Model 02 is included as Appendix A to this chapter.

For expository purposes, shadow prices determined by Optimum Model 02 will be divided into two groups. Those in the first group reflect the opportunity costs of direct substitution of equivalent

[3] Model numbers are not necessarily consecutive.

resources. For example, increasing straight time by one hour to eliminate one hour of overtime, or borrowing $1 from source A to repay source B. Shadow prices in the second group are derived from a series of operating trade-offs. An additional dollar or hour initiates a number of changes in basis variables that collectively produce the incremental profit predicted by the shadow price.

Referring to Appendix A, it should be noted that sales, ending inventory, and straight time hours (in some processes) are the only binding constraints. Overdraft funds are not binding. There are no productive opportunities for funds at 2% per period and the overdraft shadow price (1-4XLC) is properly zero. This fact is also reflected in the shadow price of cash (1-4XCA) which is equal to the interest rate savings possible if $1 of "free" cash is substituted for $1 of overdraft. The individual XCA's are equivalent to v_k* as defined by Charnes, Cooper and Miller[4] and their sum (.08) corresponds to the cumulant V_1*. Conversely, the shadow price for the factor balance (FB$) is -.02. A $1 increase in this initial amount owed the factor would reduce the opportunity to borrow in all future periods and require that an additional $1 be obtained by overdraft at a cost of $.02 per period.

There are also several less obvious dual variables in the direct substitution group. Equation (13) sets an upper limit on the number of straight time hours available in each process in every period (XT_{iT}). When all available hours are not used, e.g., 1XTM, 2XTM, etc., shadow prices are naturally zero. A positive value indicates that the stipulation is at its upper limit and reflects the opportunity cost of substitution between straight time and overtime. The derivation of variables 1-4XTC is traced in Table 5.1 and shows that the shadow price includes not only direct cost ($.5f_C$) but also the compound interest on funds that must be obtained for additional payroll.

Equation (2) is also involved in the distribution of direct labor. The shadow prices for rows M1L, SOL, C1L, and B1L, are the marginal values of one hour of "free" or supervisory labor. The interrelationship between equations (2) and (13) is direct and readily seen in the derivation of dual variables 1M1L, 1C1L, and 2SOL shown in Table 5.2.

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[4] *op. cit.*

Table 5.1
COMPA

Derivation of Shadow Prices
1 – 4XTC

4XTC:		
Overtime premium saved ($.5f_C$)		.83
Cash released, period 4	.57	
@MV = .02 (4XCA)		.0114
Shadow price check		.8414
3XTC:		
Overtime premium saved		.83
Cash released, period 3	.57	
@MV = .02 (3XCA)		.0114
Cash released, period 4	.69	
@MV = .02 (4XCA)		.0138
Shadow price check		.8552
2XTC:		
Overtime premium saved		.83
Cash released, period 2	.57	
@MV = .02 (2XCA)		.0114
Cash released, period 3	.69	
@MV = .02 (3XCA)		.0138
Cash released, period 4	.69	
@MV = .02 (4XCA)		.0138
Shadow price check		.8690
1XTC:		
Overtime premium saved		.83
Cash released, period 1	.57	
@MV = .02 (1XCA)		.0114
Cash released, period 2	.69	
@MV = .02 (2XCA)		.0138
Cash released, period 3	.69	
@MV = .02 (3XCA)		.0138
Cash released, period 4	.69	
@MV = .02 (4XCA)		.0138
Shadow price check		.8828

Table 5.2
COMPA

Derivation of Shadow Prices
1MIL, 1CIL and 2SOL

1M1L: (1XTM = 0)		
Incremental profit from 1XTM		.00
Cash released, period 1	1.23	
@MV = .02 (1XCA)		.0246
Cash released, period 2 - 4	1.32	
@MV = .06 (2-4XCA)		.0792
Shadow price check		.1038
1C1L: (1XTC = .8828)		
Incremental profit from 1XTC		.8828
Incremental cash released, period 1	1.12	
@MV = .02 (1XCA)		.0224
Incremental cash released, periods 2 - 4	1.37	
@MV = .06 (2-4XCA)		.0822
Shadow price check		.9874
2SOL: (2XTC = .69272451)		
Incremental profit from 2XTC		.69272451
Cash released, period 2	1.23	
@MV = .02 (2XCA)		.0246
Cash released, periods 2 - 4	1.32	
@MV = .04 (3-4XCA)		.0528
Shadow price check		.77012451

Economic interpretation of the dual variables (T_{jT}) is rather unusual. Because functional (10A-1) is formulated on a contribution basis, production variables have coefficients of zero. For this reason, (T_{jT}) picks up only the incremental cost of producing (one pound of product j) in period T rather than waiting until period T+1. The total marginal cost of producing one pound of product j in period T is found to be:

$$(nsp_j - gp_j) + \sum_{t=T}^{M} T_{jt}$$

This cumulative feature is apparent in Table 5.3 when dual variables 1TJT and 4TMT are verified and related to 2-4TJT and 1-3TMT.

The variables D_{jT} reflect the marginal increment to profit that will result if demand for product j can be increased by one pound in period T. This value includes the explicit margin (gp_j) plus the revenue benefits and additional costs of increased sales. To illustrate how the D_{jT} and T_{jT} variables are related, Table 5.3 also traces the derivation of 1DJT and 4DMT.

Dual variables XSJ, XSC, and XFM have more direct financial interpretations. XSJ and XSC are composed entirely of financial costs and recognize that the loss of $1 in sales revenue must be replaced by overdraft. 1XSJ is derived in Table 5.4 to illustrate how these costs interact. It should be noted that XSJ and XSC are cumulants and each reflects the total opportunity cost from T to M. Table 5.4 also shows the calculation of 1-4XFM. These variables represent the opportunity cost of reducing the maximum factor balance by $1. Factor interest saved and overdraft interest incurred offset to determine marginal cost.

The second group of dual evaluators consists of those not derived by direct and proportional substitution. In Appendix A there are only a few shadow prices in this category, specifically, 2XTS and 2-4XTB. These constraints are binding because each inequality is satisfied as an equality and slack variables are not in the basis. However, marginal revenue in these processes was not high enough to induce

Table 5.3
COMPA

Derivation of Shadow Prices
1TJT, 1DJT, 4TMT and 4DMT

1TJT:		
Labor hours not worked in period 1:		
1M1L = .0027 @MV = .1038		.00028026
1S0L = .0045 @MV = .1038		.00046710
1C1L = .00625 @MV = .9874		.00617125
1B1L = .01175 @MV = .093		.00109275
Sugar not used in period 1:		
1SUG = .0888 @MV = .08		.00710400
Raw Material not used in period 1:		
1ORM = .0781 @MV = .06		.00468600
Total saving		.01980136
Less: Incremental savings in prior periods:		
2TJT	.00073157	
3TJT	.00210329	
4TJT	.01679360	.01962846
Shadow price check		.00017290

1DJT:		
Contribution from one pound of JT:		
gp_{JT} =		.22588
Revenue from one pound of JT:		
sp_{JT} = .46129 @MV = .06370452 (1XSJ)		.029386258
Incremental profit		.255266258
Less: Incremental costs:		
1-4TJT	.01980137	
1XWJ	.0424	.06220137
Shadow price check		.193064888

Table 5.3 (continued)

4TMT:		
Labor hours not worked in period 4:		
4M1L = .0033 @MV = .8368		.00276144
4SOL = .005712 @MV = .8368		.00477980
4C1L = .006966 @MV = .8638		.00601723
4B1L = .014726 @MV = .30586263		.00450413
Sugar not used in period 4:		
4SUG = .088614 @MV = .02		.001772281
Raw Material not used in period 4:		
40RM = .09322 @MV = 0		.0
Shadow price check		.01983489

4DMT:		
Contribution from one pound of MT:		
gp_{MT} =		.192864
Revenue from one pound of MT:		
sp_{MT} = .45225 @MV = .01224224 (4XSJ)		.00553655
Incremental profit		.19840055
Less: Incremental costs:		
4TMT	.01983488	
4XWJ	.04	.05983488
Shadow price check		.13856567

Table 5.4
COMPA

Derivation of Shadow Prices
1XSJ and 1–4XFM

1XSJ:

Period 1:		
Net reduction in cash inflow:		
-.88661 @MV = .02 (1XCA)	-.0177372	
Net reduction in maximum FB:		
.392 @MV = -.00515 (1XFM)	-.0020188	
Net reduction in FB:		
-.38661 @MV = -.02 (1XFB)	.0077322	
sub-total		-.0120188
Period 2:		
Net reduction in cash inflow:		
-.88952 @MV = .02 (2XCA)	-.0177904	
Net reduction in maximum FB:		
.152 @MV = -.005425 (2XFM)	-.0008246	
Net reduction in FB:		
-.14452 @MV = -.02 (2XFB)	.0028904	
sub-total		-.0157246
Period 3:		
Net reduction in cash inflow:		
-.91286 @MV = .02 (3XCA)	-.0182572	
Net reduction in maximum FB:		
.048 @MV = -.00544 (3XFM)	-.00026112	
Net reduction in FB:		
-.03986 @MV = -.02 (3XFB)	.0007972	
sub-total		-.01772112
Period 4:		
Net reduction in cash inflow:		
-.90386 @MV = .02 (4XCA)	-.0180772	
Net reduction in FB:		
.00814 @MV = -.02 (4XFB)	-.0001628	
sub-total		-.0182400
Shadow price check		-.06370452

Table 5.4 (continued)

1XFM:	
Factor interest saved (1PFT)	.01375
Implicit interest incurred:	
.98625 @MV = -.02 (1PFT)	-.019725
Implicit interest saved:	
-.01375 @MV -.06 (1PFT)	.000825
Shadow price check	-.005150
2XFM:	
Factor interest saved (1PFT)	.01375
Implicit interest incurred:	
.98625 @MV = -.02 (1PFT)	-.019725
Implicit interest saved:	
-.01375 @MV = -.04 (1PFT)	.00055
Shadow price check	-.005425
3XFM:	
Factor interest saved (1PFT)	.014
Implicit interest incurred:	
.98625 @MV = -.02 (1PFT)	-.1972
Implicit interest saved:	
-.014 @MV = -.02 (1PFT)	.00028
Shadow price check	-.00544
4XFM:	
Factor interest saved (1PFT)	.014
Implicit interest incurred:	
.98625 @MV = -.02 (1PFT)	-.01972
Shadow price check	-.00572

the model to use overtime.[5] To illustrate what has occurred, a hypothetical marginal revenue product curve YX, for the input direct labor hours, is shown in Figure 5.1. This curve exhibits diminishing marginal product in the discontinuous jumps characteristic of linear programming.[6] Superimposed on this schedule are marginal cost curves for straight time (OS_sS_s') and overtime (S_oS_o') hours that represent those implicit in Optimum Model 02. Overtime is assumed to be available in unlimited quantities with marginal cost, S_o. Straight time is available at zero cost up to S_s hours, the point at which the curve changes from horizontal to vertical to indicate infinite cost or absolute rationing. With S_s straight time hours available, the model finds it profitable to use overtime (point a). When a total of S hours have been used marginal revenue product falls below S_oS_o' (point b) and no more overtime is demanded. If available straight time hours are increased to s_s the model will not use overtime because at point c marginal revenue is less than marginal cost.

The dual variables 2XTS and 2-4XTB are equivalent to point c in Figure 5.1. For example, in Appendix A the dual variable 2XTS is .69272451 and the marginal cost of equivalent overtime can be shown to be .8386. Therefore, although marginal revenue product is positive it is less than the marginal cost of overtime and the optimal decision is to use all available straight time hours but no overtime.

Because these dual variables are the marginal revenue product for direct labor hours rather than the opportunity cost of marginal substitutions, they cannot be verified by the procedure used in Table 5.1. The composition of variables 2XTS and 3XTB shown in Table 5.5 was determined by parametrically increasing each upper limit by one unit.

Tables 5.1 through 5.5 clearly show the extent to which the marginal cost of funds is reflected in each shadow price. For some, e.g., 1-4XTC, financial costs are a relatively minor component. However, in others, e.g., 1M1L, 1-4XSJ and 1-4XFM, financial costs constitute the total shadow price. Because in this model the marginal cost of funds is an implicit rate without counterpart in the firm, its

[5] In Appendix A, variables 2ETS and 2-4ETB are not in the optimal basis.
[6] See Baumol, *op. cit.*, pp. 291-294.

Figure 5.1
COMPA

Marginal Revenue Product,
Direct Labor Hours

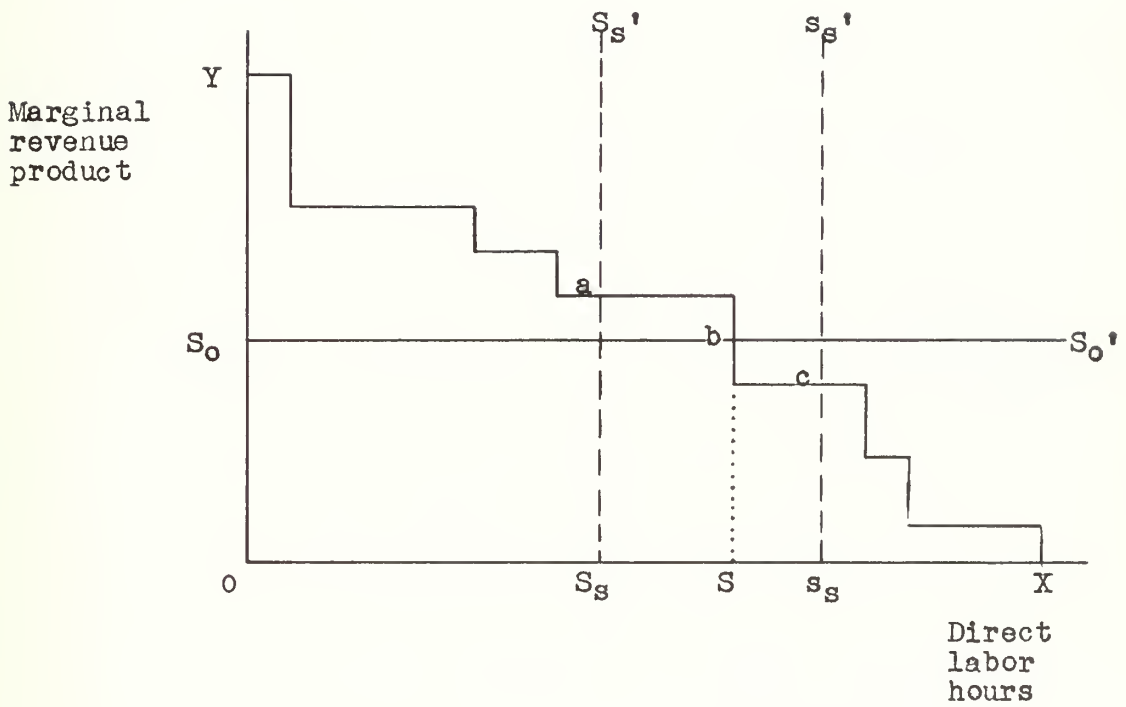

Table 5.5
COMPA

Derivation of Shadow Prices
2XTS and 3XTB

2XTS:

Variable	Change in value	(1PFT)	Extension
2PKB	290.60836486	0	0
3PKB	-290.60836486	0	0
2PBT	- 30.01065835	0	0
3PBT	30.01065835	0	0
2RTM	.78144351	0	0
3ETM	- .78144351	.8	-.625154808
2RTS	1.0	0	0
3ETS	- 1.0	.8	-.800000000
2ETC	1.71714893	.83	1.425233812
3ETC	- 1.71714893	.83	-1.425233812
2GUS	23.22130689	0	0
3GUS	- 23.22130689	0	0
2RRM	9.77380933	0	0
3RRM	- 9.77380933	0	0
2LB$	28.31446410	.02	.566289282
3LB$	9.48280382	.02	.1896560764
4LBS	- 1.17575272	.02	-.0235150544
	Shadow price check		-.6927245048

3XTB:

1PBT	- 21.29245183	0	0
3PBT	21.29245183	0	0
1RTM	- .08474396	0	0
1RTS	- .08431811	0	0
3RTB	1.0	0	0
3ETM	.08474396	.80	.067795168
3ETS	.08431811	.80	.067454488
1ETC	- .14457575	.83	-.120197873
3ETC	.14457575	.83	.120197873
1GUS	- 1.91632067	0	0
3GUS	1.91632067	0	0
1RRM	- 2.59129139	0	0
3RRM	2.59129139	0	0
1LB$	- 3.29860002	.02	-.0659720004
2LB$	- 6.26860002	.02	-.1253720004
3LB$	- 2.86687214	.02	-.0573374428
4LB$	.11158096	.02	.0022316192
	Shadow price check		-.1112001684

Note: The signs are reversed in this analysis because marginal values are calculated as reductions in cost rather than increments to profit.

impact on all shadow prices is a matter of concern. However, if an implicit source is used in the planning models, $I' > I$ is fundamental to rational decision-making.

Several objective function variations were investigated to determine if the shadow price effects of the implicit interest rate could be minimized. Most involved modifying functional (10A-1) to include various values for I' that were approximately equal to I but enough larger to permit the model to discriminate between sources of funds. At the same time, to avoid cash flow distortions, it was not desirable to change the interest rate coefficients in equation (7A). None of the implicit rate modifications proved to be particularly satisfactory because necessary tolerances in the LP-90 code confounded small differences with rounding errors and the model could not rationally discriminate between sources. Differences large enough to facilitate decision-making had shadow price effects generally as undesirable as the initial rate. In addition, it was found that in a representative linear programming model, all actual sources of external funds must be included and accurately priced if meaningful dual evaluators are to be obtained. For budgeting or planning purposes, implicit sources and costs may be included. However, if a model is to be used for both planning and shadow price analysis, implicit funds should be added to the actual source with the highest stated interest rate. This procedure will provide both the desired planning estimates and shadow prices for all non-financial stipulations that reflect the marginal cost of the highest cost actual source of funds utilized.

Possibly the most important objective of this chapter is to determine the extent to which the productive opportunity locus of funds can be defined in a real firm. In Optimum Model 02, the marginal value of cash (XCA) is the interest rate of the highest cost source of funds used. This result follows because all available funds are not required and the model is not forced to determine marginal revenue product for less than ample supplies. This schedule could be developed from point estimates

made by a series of solutions. However, it can be estimated more efficiently by using the LP-90 right-hand side parametric programming routine, DO.PLP.[7]

To illustrate the procedure, the amount of available overdraft (rows 1-4XLC in Optimum Model 02) was parametrically decreased to the point of infeasibility. That is, unless COMPA could obtain this minimum amount of funds its only alternative would be to go out of business because mandatory and predetermined funds requirements could not be satisfied.[8] Page 3 from the solution to this analysis is included as Appendix B. It shows both the minimum right-hand side value, $12,497.56, and the resulting shadow price for overdraft funds in the first period (1XLC), .4892+. The shadow price indicates that if the overdraft in all periods were limited to this level, COMPA could pay at least $1.4892 (in addition to I′) for one additional dollar of overdraft authority in period one and still break even. The dual variable 1XLC was verified by the unit increment technique used in Table 5.3 and its derivation is shown in Table 5.6. In more general terms, the company could afford to pay $1.5592592 = $1. + \sum_{t=1}^{M} XCA_t$ for one dollar that would be available for the entire planning cycle.[9] However, shadow prices are only valid in terms of the current basis and the range within which values can vary before a basis change occurs has not been defined.

The DO.PLP routine facilitates the identification of these ranges by permitting the amount of funds to be continuously increased. As more funds become available the basis will change whenever a profitable revision to the optimal decision set is possible. In addition, as funds are added, diminishing returns are evident and at each basis change (iteration) the shadow price will be less than or equal to its value in the previous basis.

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[7] C-E-I-R *op. cit.*, pp. 29-31. In the DO.PLP routine the revised right-hand side is defined as a linear function of a parameter θ, or $B = B' + \theta B''$. Where:

B' = The right-hand side in the last optimal solution.

B'' = Incremental right-hand vector used in defining the revised right-hand side.

θ = A multiplier defining the amount of B'' to be added to B' in defining B. In a parametric LP run this value varies continuously from 0 to its present maximum value.

[8] The model as formulated does not have management's flexibility to deal with this type adversity.

[9] Because no provision is made in the model for interest payments, this cost assumes that principal and interest will be paid after the current planning horizon.

Table 5.6
COMPA

Derivation of Shadow Price
1XLC

1XLC:

Variable	Change in value	(1PFT)	Extension
1PTT	17.0231629	0	0
2PTT	- 17.0231629	0	0
2PBC	- 28.12929162	0	0
3PBC	28.12929162	0	0
2RTM	- .08360025	0	0
1RTC	.10452222	0	0
1RTB	.59977710	0	0
3ETM	.08360026	.80	.066880208
2ETS	- .08351587	.80	-.066812696
3ETS	.08351587	.80	.066912696
2ETC	- .31658895	.83	-.2627688285
3ETC	.21206673	.83	.1760153859
2ETB	- .67215376	.75	-.5041152200
3ETB	.07237667	.75	.0542825025
1GUS	.32514241	0	0
2GUS	- 2.35191413	0	0
3GUS	2.02677172	0	0
1RRM	2.86567924	0	0
2RRM	- 4.53177719	0	0
3RRM	1.66609794	0	0
3FP$	2.19078189	0	0
4FP$	- 2.22188833	0	0
3FB$	- 2.22188832	.014	-.03110643648
1LB$	1.0	.02	.02000000
4LB$	- .4199124	.02	-.008398248
	Shadow price check		-.489210744408

Note: The signs are reversed in this analysis because marginal values are calculated as reductions in cost rather than increments to profit.

5.2 Introduction to the Planning Tests

Rather than proceed directly to the "what if" problems, Optimum Model 02 will be modified and used to demonstrate the test procedure and to provide a point of departure for subsequent model revisions. The model that will be used in this section is called Optimum Model 09 and consists of equations (1), (2), (3), (4), (5A), (5B), (6A), (6B), (7A), (7B), (8), (10-A), (11-1), (12-1) and (13). Equation (9) is not used and implicit funds are provided by adding $50,000 to the maximum factor balances, 1-4XFM.[10] The optimal primal-dual solution to Optimum Model 09 is included as Appendix C to this chapter.[11]

Although Optimum Model 09 is tightly constrained it retains the production, sales, and inventory options from Optimum Model 02. To illustrate how the models use these options, Table 5.7 contrasts production and overtime schedules, implicit funds requirements and profit estimates developed by Optimum Models 02 and 09 with equivalent values from Aggregate Model 02. Without a predetermined schedule both optimum models match production as closely as possible with sales and attempt to minimize intra-cycle inventories and overtime consumption. However, the substantial production changes made by Optimum Model 02 only nominally increase the level of profit. This result indicates that COMPA was doing a good job of producing to uncertain demand but more important to this analysis are the restrictive effects of limited demand that are clearly evident. The unlimited external opportunity assumptions implicit in most theoretical applications permit optimization solely in terms of resource or capacity limitations.[12] If external opportunities are also constrained the optimal decision set is sharply reduced. However, fully integrated models that realistically represent a planning cycle must include estimates of maximum and/or minimum demand.

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[10] To relax the maximum factor balance the variables $FB\$_T$ must be increased. Because these variables have negative signs in equation (7B), the $50,000 must be subtracted from the right-hand side.

[11] This is the last complete solution that will be included in this chapter.

[12] For example, the warehouse model examples in Charnes, Cooper, and Miller, *op. cit.*

Table 5.7
COMPA

Comparison of Plans and Estimates Developed by
Aggregate Model 02, and Optimum Models 02, 09, and 09-1

	Aggregate Model 02	Optimum Models: 02	09	09-1
Contribution (1PFT)	$ 266,285	$ 267,343	$ 267,629	$ 266,131
Implicit funds supplied:				
Period:1	$ 25,480	$ 19,606	$ 20,585	$ 11,672
2	24,320	12,107	17,598	(1,824)
3	16,841	2,406	4,480	(9,072)
4	17,934	13,466	14,152	9,590
Overtime manhours used:				
Process: M1L				
Period: 1	105	(15)	0	(199)
2	(35)	(102)	0	(124)
3	162	122	6	205
4	255	482	482	605
Process: S0L				
Period: 1	147	(57)	(27)	(248)
2	37	0	119	(24)
3	262	190	41	282
4	184	496	496	619
Process:C1L				
Period: 1	682	226	266	(92)
2	879	691	907	643
3	808	855	599	974
4	866	1,463	1,463	1,710
Process: B1L				
Period: 1	(531)	(4)	(4)	(2,102)
2	(45)	0	0	(31)
3	56	0	0	0
4	516	0	0	1,667
Production Schedule: (#)				
Period 1, Product:				
JT	59950	40439	49091	40439
ST	57742	20431	20431	20431
SJ	0	0	0	0
KB	145944	129042	129042	129042
KT	4360	1241	1241	1241
TT	4228	0	0	0
BC	218167	200599	200599	200599
BT	19805	55977	53813	12293
MC	14550	5160	5160	5160
MT	0	3144	3144	0
Total, period 1	524746	456033	462021	409205

Table 5.7 (continued)

Production Schedule: (#)				
Period 2, Product:				
JT	91832	97539	88887	97539
ST	76542	94846	94846	94846
SJ	0	0	0	0
KB	185542	189054	232324	179862
KT	2508	4405	4405	4405
TT	10153	4909	4909	4909
BC	257065	212012	212012	212012
BT	31911	32615	30311	32150
MC	15545	16595	16595	16595
MT	3310	0	0	2372
Total, period 2	674408	651975	684289	644690
Period 3, Product:				
JT	85578	83964	86964	83964
ST	89353	85063	85063	85063
SJ	0	0	0	0
KB	190152	162123	118853	171315
KT	3290	4164	4164	4164
TT	11253	25155	25155	19177
BC	187885	219495	219495	219495
BT	29462	18806	23275	31959
MC	17605	20152	20158	20130
MT	166	0	0	772
Total, period 3	614744	618928	583127	636039
Period 4, Product:				
JT	55153	70571	70571	70571
ST	33327	56624	56624	56624
SJ	57531	57531	57531	57531
KB	165256	206675	206675	206675
KT	2384	2732	2732	2732
TT	4430	0	0	5978
BC	214017	245028	245028	245028
BT	26221	0	0	30997
MC	28000	33787	33787	33815
MT	0	332	332	332
Total, period 4	586319	673280	673280	710283

Comparing Optimum Models 02 and 09 shows that although the plans proposed by these models are similar, there are a number of significant differences. These variations must be attributed to the lower marginal cost of funds (1-4XCA) in Optimum Model 09 and they emphasize why the inclusion of implicit interest rates is a matter of concern. COMPA cannot obtain more funds from its factor but neither does it have a source at 2% per period. The factor is the highest cost actual source of funds and decisions should be based on analyses incorporating this interest rate.

In addition to providing the operating plan summarized in Table 5.7, Optimum Model 09 can be used to obtain the marginal revenue product curve for cash. Simply stated, the procedure used to trace this schedule first quantifies the minimum feasible input of external funds and next continuously increases the supply until the marginal value falls to zero. This procedure is applied to Optimum Model 09 by first reducing the maximum factor balances (1-4XFM) in all periods to a joint minimum. Appendix C, page C-5, shows the shadow prices and right-hand side values for 1-4XFM after this parametric routine has been completed. The non-zero shadow price for 1XFM indicates that maximum demand occurs in period 1 when $12,671.79 is required to maintain feasibility.

The dual evaluator 1XFM is the marginal value of a $1 increase in the maximum factor balance. This variable is analogous to 1XLC in Optimum Model 02 and it can be seen by reference to Table 5.6 that the marginal value derives from overtime savings in periods 2 and 3 permitted by the ability to use more straight time in period 1. However, this result follows because there are ample funds available in periods 2 through 4. At current levels the factor balance includes nonexistent or implicit funds and there is no guarantee that they will be supplied when required. For this reason, the value of funds limited to minimum levels in every period provides a more meaningful shadow price.

To obtain the individual values needed to quantify this cumulative variable requires a series of parametric programming routines. First, the right-hand side vector 1-4XFM is set at the joint minimum determined in Optimum Model 09 and 2XFM is minimized. The vector is then revised to reflect the new value for 2XFM and 3XFM is minimized. Finally, with 1-3XFM set at their individual

minimum feasible levels, 4XFM is minimized. The individual minimum values for 1-4XFM and the shadow prices derived from this analysis are shown in Appendix C, page C-6.

The optimal production schedule and other plans that the model developed to cope with the stringent funds rationing imposed by parametric minimization are shown in column 09-1 of Table 5.7. Comparing these values with equivalent decisions from Optimum Model 09 indicates how the power of linear programming analysis can conserve cash without materially reducing profit. Although sales are not included in Table 5.7 the complete solution to Optimum Model 09-1 shows that the model did not reduce total production or forgo possible sales to conserve cash. This result must be attributed to the factoring arrangement that converts sales into instant cash.

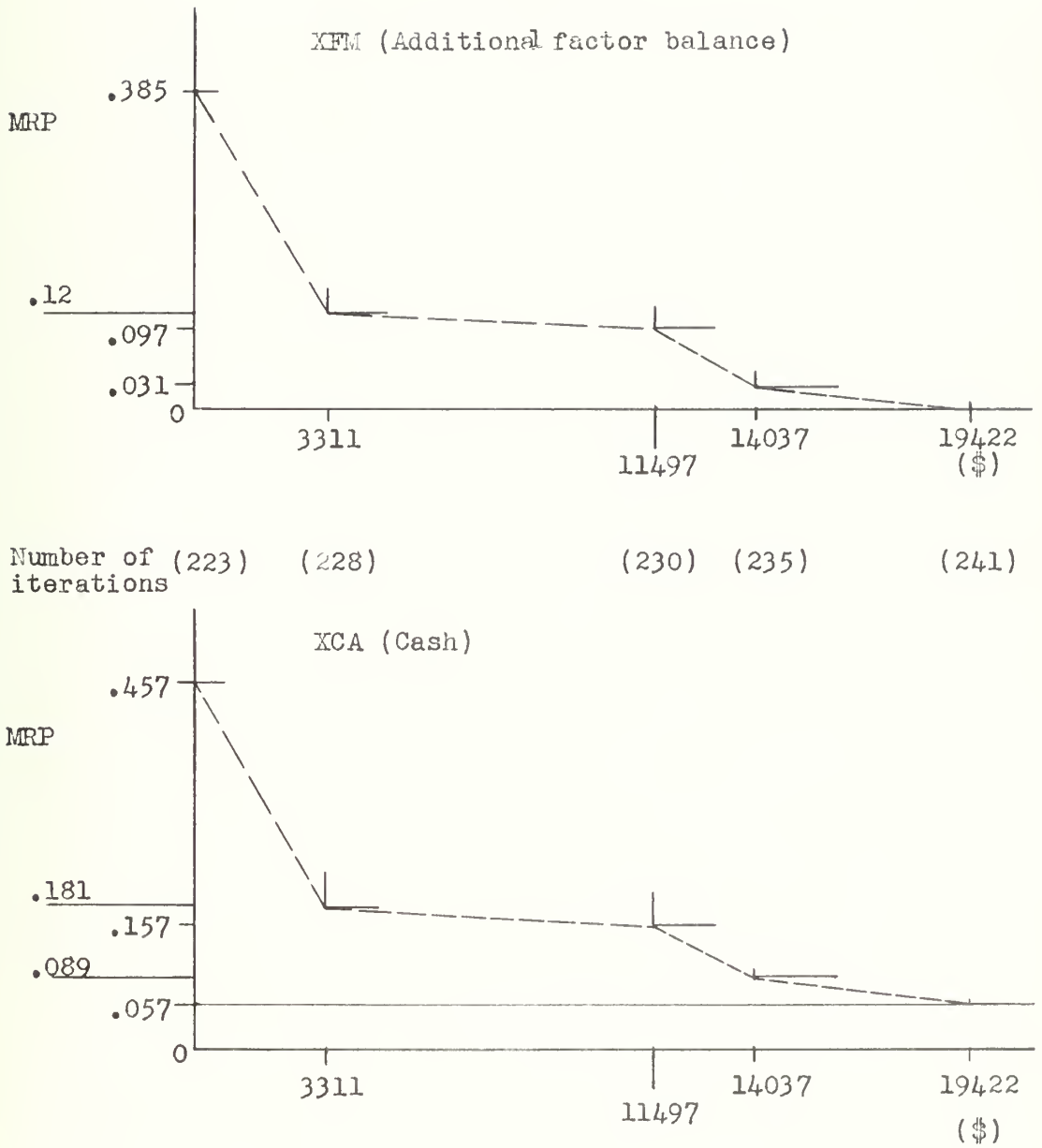
To return to the opportunity cost of funds, the marginal revenue schedule will be traced by continuously increasing the maximum factor balance 1-4XFM until the cumulant, $\sum_{t=1}^{M} XFM_t = 0$. The cumulants 1-4XFM and 1-4XCA obtained by applying this procedure to Optimum Model 09-1 are plotted in Figure 5.2. The marginal revenue product curves shown in Figure 5.2 have the characteristic discontinuities although a technical shortcoming makes it impossible to completely define the schedule and join the steps. Because it is impractical to obtain the basis at each iteration there are a series of undefined steps between each of the points that have been plotted. The iteration count row is included in Figure 5.2 to show the number of basis changes between observations. This count indicates that the complete marginal revenue product curve in this analysis could have up to 19 steps. To emphasize the general shape of the curve the observed values have been connected by a broken line. The sharp initial drop in opportunity cost is typical of those traced in other analyses using realistically constrained models. In addition to diminishing returns this reflects the absence of unlimited external opportunities.

5.3 The Four Period Planning Models

The first series of planning tests involves the formulation and solution of a series of closely related "what if" problems. The models that will be used are similar to Optimum Model 09 and represent the

Figure 5.2
COMPA

Optimum Model 09
Marginal Revenue Product,
XFM and XCA



July–October 1966 planning cycle. Test procedures will follow the example in section 5.2. First, the fully integrated model will be modified to incorporate the planning assumptions. Second, it will be solved assuming an unlimited supply of external funds at a specified cost. Third, available funds will be reduced parametrically to minimum feasible levels to determine the impact of funds rationing on profit and operating decisions. Fourth, demand will be permitted to increase by a maximum of 10% to evaluate the model's response and the change in funds requirements. In addition, a fifth step will be added in some of the tests to trace the marginal revenue curves for cash.

Problems that will be considered are hypothetical and have been devised to test the usefulness of the fully integrated model for planning purposes. However, they are based on actual situations in COMPA and the alternatives that will be evaluated are reasonable responses to specified areas of concern.

COMPA normally schedules production to very short term sales forecasts or to orders booked. This scheduling procedure is necessitated by the prevalent cash shortage and its objective is to minimize investment in finished goods inventory. Stockouts are a constant problem with this system and at least 10% of all orders must be canceled because they cannot be filled in time to comply with requested shipping dates. Since orders canceled are potentially profitable incremental business COMPA naturally wants to eliminate stockouts. To do this will require that minimum finished goods inventories be maintained at all times. However, management has not been able to determine the impact on overall operations if scarce funds are used to implement this policy.

A second problem is the need to stabilize employment. Producing to sales in a seasonal business causes severe dislocations in the work force. In slow periods many workers can be laid off but key employees and foremen must be retained. During peak seasons new workers usually cannot be added as rapidly as desired and their initial productivity is always low. For this reason an inordinate amount of overtime must be incurred in peak months if all possible orders are to be filled. To stabilize the work force management has considered reducing seasonality by scheduling production to annual sales forecasts.

Level production would eliminate layoffs, minimize overtime expense, and increase productivity. However, it would also involve the accumulation of large amounts of finished goods inventory during the off-peak months. Here again management is uncertain how much additional financing would be required to adopt this policy and what its impact would be on overall profitability.

During 1966 the factor was COMPA's only significant source of external funds. However, management was continually seeking a term loan to refinance tax liens, overdue trade accounts and other short-term obligations. The amount needed to settle overdue accounts can be readily determined from company records but uncertain permanent working capital requirements cannot be defined. Prolonged financial adversity has caused both permanent and seasonal needs to be satisfied from short-term sources and available information does not indicate the level of payments directly associated with current operations. For this reason better estimates of short-term funds requirements are needed to verify the amount of long-term financing that should be sought.

A joint analysis of these three problem areas requires six planning models. These models will be identified as 01 through 06 and each will include most of the basic equations developed in Chapter 4. However, a few equations must be modified to incorporate specific assumptions and new values will also be needed for several coefficients and right-hand side vectors. To facilitate the discussion, Table 5.8 indicates the assumptions included in each of the six models.

A minimum intra-cycle inventory assumption is included in all models to represent one possible answer to the stockout problem. This stipulation represents a policy decision by management to maintain finished goods inventories equal to or larger than expected demand during the next two-week period. Including this policy in the models requires that equations (11) and (11-1) be revised as follows:

$$B_{jT} \geq .5S_{jT+1}$$

or

$$B_{jT} - .5S_{jT+1} \geq 0 \qquad \begin{matrix} T = 1,M-1 \\ j = 1,N \end{matrix} \qquad (11\text{-}2)$$

and

$$B_{jT} \geqslant F_{jT} \qquad \begin{matrix} T = M \\ j = 1,N \end{matrix} \qquad (11\text{-}2A)$$

Equation (11-2) functionally relates inventory at the end of period T to expected sales in T+1 and the coefficient .5 approximates a two-week minimum stock level. The Type-II inequalities in both expressions establish lower limits. Inequality (11-2A) is an ending condition and it recognizes that sales in period M+1 are unknown to the model. The right hand side values ($4F_j$) used in the planning models are slightly different from those in Optimum Model 09 and in this case are actual counts taken from COMPA's October 31, 1966, inventory report. Revised values for $4F_j$ are shown in Appendix D.

Table 5.8
COMPA
Summary of Planning Assumptions

	Long Term Financing					
	Factor		Term Loan			
Planning Model:	01	02	03	04	05	06
Minimum intra-cycle inventory	x	x	x	x	x	x
Workforce:						
"Ideal"	x			x	x	
Determined by model		x	x			x
Short-term financing:						
Factor	x	x	x	x		
Line of credit					x	x

The proposed stable employment or level production policy will be evaluated in two ways. Models 01, 04, and 05 are provided with labor inputs to the various processes that represent the "ideal" size stable work force visualized by management. As a check on the optimality of management's estimates, models 02, 03 and 06 are constrained to determine a comparable stable work force for the four periods in the analysis. Forcing the planning models to level production does not imply that the same amount of each item will be produced in every period. Rather, the objective is optimal utilization of a

constant supply of labor and the extent to which this also smoothes production depends on the distribution of sales.

Including the stable work force stipulations requires that idle time variables (UT_{iT}) be used for the first time. These variables are added primarily to inform the models that unused manhours have a positive cost. Because specific quantities are stipulated, straight time manhours should be used to the maximum extent possible before overtime is scheduled in any period. If all available straight time hours cannot be profitably employed, appropriate UT_{iT} vectors will appear in the optimal basis. To incorporate the UT_{iT} variables in the models requires that additional terms be included in the functional (10A-1) and equations (2) and (8). Because financing assumptions also effect equation (8), the revised form (8-1) is used in 01 and 02 only.

$$
\begin{aligned}
& R_M \left(\sum_{t=1}^{T-1} RTM_t + \sum_{t=1}^{T-1} UTM_t \right) + w_M R_M (RTM_T + UTM_T) \\
& + R_S \left(\sum_{t=1}^{T-1} RTS_t + \sum_{t=1}^{T-1} UTS_t \right) + w_S R_S (RTS_T + UTS_T) \\
& + R_C \left(\sum_{t=1}^{T-1} RTC_t + \sum_{t=1}^{T-1} UTC_t \right) + w_C R_C (RTC_T + UTC_T) \\
& + R_B \left(\sum_{t=1}^{T-1} RTB_t + \sum_{t=1}^{T-1} UTB_t \right) + w_B R_B (RTB_T + UTB_T) \qquad (8\text{-}1) \\
& + R_M' \sum_{t=1}^{T-1} ETM_t + w_M R_M' (ETM_T) + R_S' \sum_{t=1}^{T-1} ETS_t + w_S R_S' (ETS_T) \\
& + R_C' \sum_{t=1}^{T-1} ETC_t + w_C R_C' (ETC_T) + R_B' \sum_{t=1}^{T-1} ETB_t + w_B R_B' (ETB_T) \\
& + \sum_{t=1}^{T} GUS_t + \sum_{t=1}^{T-1} RRM_t + K \sum_{t=1}^{T-1} WGJ_t + K' \sum_{t=1}^{T-1} WGC_t + m \sum_{t=1}^{T-1} SJ\$_t
\end{aligned}
$$

$$-x\sum_{t=1}^{T} SJ\$_t - x'\sum_{t=1}^{T} SC\$_t - (y)(e_T)SJ\$_T - (y)(e_{T-1})SJ\$_{T-1}$$

$$-(y)(e_{T-2})SJ\$_{T-2} - (y)(e_{T-3})SJ\$_{T-3} - (y)(e_{T-4})SJ\$_{T-4}$$

$$-e_T'(SC\$_T) - e_{T-1}'(SC\$_{T-1}) - e_{T-2}'(SC\$_{T-2}) - e_{T-3}'(SC\$_{T-3})$$

$$-e_{T-4}'(SC\$_{T-4}) + \sum_{t=1}^{T} FP\$_t + CA\$_T = CA\$_0 - \sum_{t=1}^{T} NMR_t$$

$$T = 1,M$$

The revised functional has the form:

$$\text{maximize } \pi = \sum_{t=1}^{M} GP\$_t - .5f_M \sum_{t=1}^{M} ETM_t - .5f_S \sum_{t=1}^{M} ETS_t$$

$$-.5f_C \sum_{t=1}^{M} ETC_t - .5f_B \sum_{t=1}^{M} ETB_t - f_M \sum_{t=1}^{M} UTM_t$$

$$-f_S \sum_{t=1}^{M} UTS_t - f_C \sum_{t=1}^{M} UTC_t - f_B \sum_{t=1}^{M} UTB_t - K \sum_{t=1}^{M} WGJ_t \qquad (10A\text{-}2)$$

$$-K' \sum_{t=1}^{M} WGC_t - I \sum_{t=1}^{M} FB\$_t - I' \sum_{t=1}^{M} LB\$_t$$

In functional (10A-2) idle time is twice as costly as overtime. Although overtime is expensive its use only increases production cost by the amount of premium. On the other hand, idle time produces nothing and its total cost is wasted.

Equation (2) has the following form when revised:[13]

$$\sum_{j=1}^{N} 1_{ij}P_{jT} - RT_{iT} - ET_{iT} + UT_{iT} = FT_{iT} \qquad \begin{matrix} T = 1,M \\ i = M1L, SOL, C1L, B1L \end{matrix} \qquad (2\text{-}1)$$

[13] In 01, equation (2) was used and the variable UT_{iT} was included in (13). This formulation was abandoned for all subsequent models to facilitate interchange of the ideal and determined workforce assumptions. Models 04 and 05 use equations (2-1) and (13-1).

In (2-1) a positive difference $\sum_{j=1}^{N} l_{ij}P_{jT} - RT_{iT}$ represents a need for overtime and it is picked up by ET_{iT}. If supply exceeds demand it is reflected in UT_{iT}. The right-hand side values, FT_{iT} that will be used in the planning models represent management's "ideal" supervisory force expressed in total manhours per period in each process. These values are shown in Appendix D.

"Ideal" straight time manhours (RT_{iT}) are stipulated by constraint (13). In models 01, 04, and 05 it is changed to an equality and has the form:

$$RT_{iT} = XT_{iT} \qquad (13\text{-}1)$$

$$T = 1,M$$

$$i = M1L, SOL, C1L, B1L$$

The XT_{iT} values reflect management's estimate of the number of direct employees in the ideal work force expressed in manhours per period. Quantities are shown in Appendix D.

Forcing the model to determine an optimal number of straight time manhours across all periods requires a different procedure. In models, 01, 03 and 06, equation (13) has the form:

$$RT_{iT} = q_T RT_{iT+1}$$

or

$$RT_{iT} - q_T RT_{iT+1} = 0 \qquad (13\text{-}2)$$

$$T = 1, M-1$$

$$i = M1L, SOL, C1L, B1L$$

Where:

q_T = The ratio of working days in period T to those in period T+1. This coefficient forces the model to recognize that total monthly manhours must reflect both hours per day and the actual number of days worked. The coefficient values are:

$q_1 = 20/23 = .869565$

$q_2 = 23/21 = 1.095238$

$q_3 = 21/20 = 1.05$

The assumptions concerning short-term financing are combined in three ways. Models 01 and 02 continue to use the factor as the only source of external funds and the right-hand side values shown in Table 4.10 are used in equation (8-1). Models 03 and 04 are formulated to reflect the assumption that

all overdue current liabilities have been refinanced by a term loan and that during the period all payments are made in accordance with stated credit terms. However, all short-term or operating funds continue to be supplied by the factor. Incorporating this assumption requires that equation (8-1) be modified to have the form:

$$
\begin{aligned}
& zf_M \left(\sum_{t=1}^{T-1} RTM_t + \sum_{t=1}^{T-1} UTM_t \right) + w_M R_M (RTM_T + UTM_T) \\
& + zf_S \left(\sum_{t=1}^{T-1} RTS_t + \sum_{t=1}^{T-1} UTS_t \right) + w_S R_S (RTS_T + UTS_T) \\
& + zf_C \left(\sum_{t=1}^{T-1} RTC_t + \sum_{t=1}^{T-1} UTC_t \right) + w_C R_C (RTC_T + UTC_T) \\
& + zf_B \left(\sum_{t=1}^{T-1} RTB_t + \sum_{t=1}^{T-1} UTB_t \right) + w_B R_B (RTB_T + UTB_T) \\
& + z(1.5f_M) \sum_{t=1}^{T-1} ETM_t + w_M R_M'(ETM_T) + z(1.5f_S) \sum_{t=1}^{T-1} ETS_t \\
& + w_S R_S'(ETS_T) + z(1.5f_C) \sum_{t=1}^{T-1} ETC_t + w_C R_C'(ETC_T) \qquad (8\text{-}2) \\
& + z(1.5f_B) \sum_{t=1}^{T-1} ETB_t + w_B R_B'(ETB_T) + \sum_{t=1}^{T} GUS_t + \sum_{t=1}^{T-1} RRM_t \\
& + K \sum_{t=1}^{T-1} WGJ_t + .75K(WGJ_T) + K' \sum_{t=1}^{T-1} WGC_t + .75K'(WGC_T) \\
& + m \sum_{t=1}^{T-1} SJ\$_t - x \sum_{t=1}^{T} SJ\$_t - x' \sum_{t=1}^{T} SC\$_t + (y)(e_T)SJ\$_T - (y)(e_{T-1})SJ\$_{T-1}
\end{aligned}
$$

$$- (y)(e_{T-2})SJ\$_{T-2} - (y)(e_{T-3})SJ\$_{T-3} - (y)(e_{T-4})SJ\$_{T-4}$$

$$- e_T'(SC\$_T) - e_{T-1}'(SC\$_{T-1}) - e_{T-2}'(SC\$_{T-2}) - e_{T-3}'(SC\$_{T-3})$$

$$- e_{T-4}'(SC\$_{T-4}) + \sum_{t=1}^{T} FP\$_t + CA\$_T = CA\$_0 - \sum_{t=1}^{T} NMR_t$$

$$T = 1,M$$

Where:

z = The total cash flow per manhour worked. This coefficient functionally relates employers' F.I.C.A. payments to hours worked. z = 1.042.

In equation (8-2) the direct labor cost coefficient (f_i) has been substituted for the net cash flow coefficients R_i and R_i' in the cumulative payroll terms. This change transfers payroll withholdings to IRS in the month due and it also functionally relates these payments to direct labor hours worked. The coefficient (z) adds the employer's share of F.I.C.A. to the total cash flow. The terms $.75K(WGT_T)$ and $.75K'(WGC_T)$ are added to force payment of freight charges in accordance with the one week credit terms.

The final financing alternative is included in models 05 and 06. This option also incorporates the assumption that all overdue obligations have been refinanced and that payments are on a current basis. However, these models also assume that the fictional term loan was large enough to liquidate the initial factor balance ($175,100) and that short-term funds are now obtained through a line of credit arrangement. Incorporating this set of assumptions in models 05 and 06 requires removing equations (7A) and (7B) and column vectors 1-4FB$ and 1-4FP$. In addition coefficients x and x' are set equal to zero and inequality (9) and variables 1-4LB$ are reactivated. Equation (8-2) must also be modified to eliminate the FP$ term and add the line of credit.

$$\ldots\ldots - LB\$_T + I' \sum_{t=1}^{T-1} LB\$_t \ldots\ldots \qquad (8\text{-}3)$$

$$T = 1,M$$

These terms reflect both the source of funds and the payment of interest. The coefficient I' in (8-3) and (10A-2) reflects the assumed rate applicable to the line of credit, or $I' = .012$. The same right-hand side vector is used with both equations (8-2) and (8-3) and reflects net non-model related cash flows assuming all obligations are paid when due. These values also recognize that several additional flows have been related to matrix variables in (8-2) and (8-3). Calculation of the right-hand side values used in models 03 through 06 is shown in Table 5.9. All individual flows were estimated from COMPA's records to represent what actual payments would have been under the assumed financing arrangements. The initial overdraft is included because it seems reasonable that this deficit should be replaced by short-term rather than long-term funds.

All modifications required to incorporate various aspects of the "what if" problems into the six planning models have been explained. Some of this discussion has been rather complex and to allay confusion the composition of all COMPA aggregate models formulated thus far in the study is summarized in Table 5.10. It should also be mentioned that actual sales (Appendix I, Chapter 4) are specified as upper limits on demand in all planning models. Optimization based on historical sales is equivalent to planning under certainty.

5.4 Evaluation of the Four Period Planning Models

The COMPA four period planning models are large linear programming problems in absolute terms but quite manageable relative to the comprehensive model developed in Chapter 4. Model dimensions vary slightly, depending on the specific assumptions included, but average size is about 200 rows by 300 columns (including slacks).

All planning tests have followed the procedure outlined earlier in this chapter and a series of solutions has been obtained for each model. The four period planning horizon is too short to permit the models to develop plans that could be adopted without reservation. However, extending the

Table 5.9
COMPA

Calculation of Right Hand Side Values,
Equations (8-2) and (8-3)

	Period (1)	(2)	(3)	(4)
Cash outflows:				
Mandatory payments, T	$ 13,699	$ 17,669	$ 13,942	$ 14,211
Misc. and net payroll, T	46,246	39,648	48,263	38,333
Freight, ¼ of June	4,649	0	0	0
Div. 1 sales com., June	4,133	0	0	0
NMM payments, accrued T-1:				
Misc exp., Div. 1	1,124	1,296	2,560	1,726
Adm. exp., factory	1,151	614	755	690
Overhead, factory	4,700	4,989	5,076	5,598
Misc exp., factory	322	140	(1,000)	1,301
Legal and audit	0	400	0	0
Withholding	6,762	8,184	6,970	8,412
Employers FICA	1,717	1,989	1,629	1,884
Other raw material, June	35,463	0	0	0
To subsidiaries	2,400	0	1,500	1,500
Initial cumulative cash deficit	44,671	0	0	0
Total outflows	$167,007	$ 74,929	$ 79,695	$ 73,655
Inflows and adjustments:				
Misc. receipts	$ 831	$ 913	$ 827	$ 760
[1]Direct portion, net payroll	11,250	13,600	17,450	14,555
Direct withholding	0	2,678	3,174	4,262
Direct portion, employers FICA	0	643	777	1,023
From left hand side	114,431	53,712	43,075	9,208
Total adjustments	$126,512	$ 71,446	$ 65,303	$ 29,808
Net outflow	$ 40,495	$ 3,483	$ 14,392	$ 43,847
Cumulative net outflow	$ 40,495	$ 43,978	$ 58,370	$102,217

1, Calculation of direct portion, net payroll is shown in Table 4.10.

Table 5.10
COMPA

Composition of Aggregate Models

Equation number:	AM	Optimum		Planning					
	02	02	09	01	02	03	04	05	06
1	x	x	x	x	x	x	x	x	x
2	x	x	x	x					
2-1					x	x	x	x	x
3	x	x	x	x	x	x	x	x	x
4	x	x	x	x	x	x	x	x	x
5A	x	x	x	x	x	x	x	x	x
5B	x	x	x	x	x	x	x	x	x
6A	x	x	x	x	x	x	x	x	x
6B	x	x	x	x	x	x	x	x	x
7A	x	x	x	x	x	x	x		
7B	x	x	x	x	x	x	x		
8	x	x	x						
8-1				x	x				
8-2						x	x		
8-3								x	x
9	x	x						x	x
10A-1	x	x	x						
10A-2				x	x	x	x	x	x
10B-1	x	x	x						
10C	x	x	x	x	x	x	x	x	x
10D	x	x	x						
11	x								
11-1		x	x						
11-2				x	x	x	x	x	x
11-2A				x	x	x	x	x	x
12	x								
12-1		x	x	x	x	x	x	x	x
13	x	x	x						
13-1				x			x	x	
13-2					x	x			x

models to include additional periods makes them more difficult to discuss and their illustrative value is diminished. In addition, many of the parametric techniques place heavy demands on computer resources and more variations can be considered if smaller models are tested. In the next section a few ten period models will be developed to demonstrate that the concepts and procedures can be applied to a more realistic planning horizon.

Solutions to planning models 01 through 06 in LP-90 format will not be reproduced in this paper. Complete computer printouts of these solutions cover approximately 600 pages and could fill a sizable volume. In addition, tracing specific variables, shadow prices, etc., through the various tests would soon become a wearisome task for the most industrious reader. Important decision variables and other information from the six planning problems and Optimum Model 09 are compared in Table 5.11. Section (a) of this table shows plans and estimates developed by the models when adequate funds are provided. In section (b), equivalent observations are contrasted after the models have been forced to reduce implicit fund requirements to minimum feasible levels. The impact of a 10% increase in demand is reflected in section (c) and straight time and overtime labor requirements that correspond to the situations in (a), (b), and (c) are shown in sections (d), (e), and (f).

A meaningful analysis of the test results must translate the mass of data shown in Table 5.11 into a few significant generalizations that summarize what has been learned about the usefulness of linear programming for short-term financial planning. For this reason the discussion will concentrate on interpretation rather than derivation. Examples were provided earlier in this chapter to illustrate the derivation and definition of specific values and this type of detailed analysis will not be repeated here.

In the first test it was assumed that COMPA has access to an adequate supply of short-term funds. Availability of ample funds permits optimal production schedules to be developed in response to known demand and specified operating restrictions. Under these conditions the fully integrated models minimize financing cost but their power to optimally allocate limited funds is not utilized.

Table 5.11
COMPA

Comparison of Plans and Estimates Developed by
Optimum Model 09 and Planning Models 01 through 06

	Optimum Model 09	Planning Model: 01	02	03	04	05	06
(a.) Adequate funds:							
Contribution (1PFT)	$ 267,629	$ 262,045	$ 267,299	$ 268,160	$ 262,894	$ 273,334	$ 278,828
Marginal value of cash	.0575	.0575	.0575	.0575	.0575	.0488	.0488
Implicit funds supplied:							
Period: 1	$ 20,585	$ 49,736	$ 47,979	$ 52,040	$ 53,796	$ 58,154	$ 55,970
2	17,598	60,930	66,983	46,754	41,240	59,589	73,484
3	4,480	49,545	70,215	43,740	23,380	39,515	82,151
4	14,152	44,344	81,304	62,155	25,202	21,883	94,127
Production schedule: (#)							
Period 1, Product: JT	49091	89208	-	-	-	-	-
ST	20431	67854	-	-	-	-	-
SJ	0	0	0	0	0	0	0
KB	129042	218923	-	-	-	-	-
KT	1241	3444	-	-	-	-	-
TT	0	1659	-	-	-	-	4954
BC	200599	306605	-	-	-	-	-
BT	53813	28368	-	-	-	-	-
MC	5160	13458	-	-	-	-	-
MT	3144	1142	-	-	-	-	-
Total, period 1	462021	730711	730711	730711	730711	730711	734006
Period 2, Product: JT	88887	90752	-	-	-	-	-
ST	94846	89954	-	-	-	-	-
SJ	0	0	0	0	0	0	0
KB	232324	175588	-	-	-	-	-
KT	4405	4284	-	-	-	-	-
TT	4909	12838	-	-	-	-	9544
BC	212012	215754	317358	-	215754	-	414350
BT	30311	32054	-	-	-	-	-
MC	16595	18362	-	-	-	-	-
MT	0	1616	-	-	-	-	-
Total, period 2	684289	641202	742806	742806	641202	641202	836504

Table 5.11 (continued)

(a.)								
Period 3, Product:	JT	83964	71183	-	-	-	-	-
	ST	85063	62141	-	-	-	-	-
	SJ	0	0	0	0	-	-	-
	KB	118853	152798	170405	-	152798	-	190667
	KT	4164	3125	-	-	-	-	-
	TT	25155	10010	-	-	-	-	-
	BC	219495	208918	276488	-	208918	-	351415
	BT	23275	23520	26477	-	23520	-	24508
	MC	20158	25108	-	-	-	-	-
	MT	0	432	5639	-	432	-	-
Total, period 3		583127	585309	678650	678650	585309	585309	766663
Period 4, Product:	JT	70571	56196	-	-	-	-	-
	ST	56624	29484	-	-	-	-	-
	SJ	57531	79256	-	-	-	-	-
	KB	206675	137772	127354	-	137772	-	146652
	KT	2732	2516	-	-	-	-	-
	TT	0	624	-	-	-	-	-
	BC	245028	179816	282604	-	179816	-	353963
	BT	0	16218	32512	-	16218	-	30636
	MC	33787	19986	-	-	-	-	-
	MT	332	11367	10066	-	11367	11367	5107
Total, period 4		673280	533235	640598	640598	533235	533235	724420
Excess inventory, end of period 4, product:								
	KB	0	0	7190	-	0	0	46750
	BC	0	0	271962	-	0	0	515240
	BT	0	0	19251	-	0	0	15405
	MT	0	10748	14654	-	10748	10748	4488
Total		0	10748	313057	313057	10748	10748	581883

(b.)Minimum funds:

Contribution (1PFT)	$ 266,131	$ 248,868	$ 252,259	$ 231,935	$ 235,458	$ 157,427	$ 127,459
Percentage decrease in contribution from (a.)	0.5	5.0	5.5	13.5	10.5	42.3	54.3
Marginal value of cash	.457218	47.7684	142.9397	92.2957	235.783	90.5973	298.5574
Implicit funds supplied:							
Period: 1	$ 12,672	$ 51,283	$ 50,777	$ 45,435	$ 45,949	$ 31,637	$ 17,166
2	(1,824)	55,780	54,853	45,435	45,949	31,637	16,799
3	(9,072)	55,780	54,853	45,435	45,949	25,351	(2,608)
4	9,590	55,780	54,853	45,435	45,949	31,637	17,166

Table 5.11 (continued)

(b.)								
Production schedule: (#)								
Period 1, Product:	JT	40439	89208	-	-	-	89208	53466
	ST	20431	67854	-	-	-	20431	20431
	SJ	0	0	0	0	0	0	0
	KB	129042	218973	-	-	-	129042	95688
	KT	1241	3444	-	-	-	3444	-
	TT	0	1659	-	-	-	1659	0
	BC	200599	306605	-	224965	238713	176834	0
	BT	12293	28368	-	-	27524	28368	12293
	MC	5160	0	0	5160	-	5160	0
	MT	0	1142	-	-	-	1142	0
Total, period 1		409205	717253	717253	640773	648517	455288	185322
Period 2, Product:	JT	97539	90752	-	-	-	90752	64510
	ST	94846	89954	-	-	-	42532	46217
	SJ	0	0	0	0	0	0	0
	KB	179862	175588	-	-	-	74673	89166
	KT	4405	4284	-	-	-	4284	4284
	TT	4909	12838	-	-	-	22848	0
	BC	212012	175035	180043	43127	67027	0	0
	BT	32150	32054	-	-	31211	32054	9279
	MC	16595	8298	-	0	0	10065	0
	MT	2372	1616	-	-	-	1616	907
Total, period 2		644690	590419	595427	450213	473270	278824	214363
Period 3, Product:	JT	83964	71183	-	-	-	-	61682
	ST	85063	62141	-	-	-	-	38846
	SJ	0	28074	-	-	-	-	0
	KB	171315	152798	127530	152798	-	74673	82149
	KT	4164	3125	-	-	-	-	-
	TT	19177	10010	-	-	-	0	0
	BC	219495	135142	173208	101461	114592	0	0
	BT	31959	23520	-	-	-	-	9279
	MC	20130	10105	15042	13019	12414	10065	0
	MT	772	432	-	-	-	-	0
Total, period 3		636039	496530	514265	466763	478289	283213	195081
Period 4, Product:	JT	70571	56196	-	-	-	-	-
	ST	56624	29484	-	-	-	-	9874
	SJ	57531	79255	-	-	-	-	51181
	KB	206675	137772	112504	137772	-	70632	-
	KT	2732	2516	-	-	-	-	-
	TT	5978	624	21437	7094	7809	31373	203
	BC	245028	146757	179816	163344	166326	80645	-
	BT	30997	16218	-	-	-	-	8677
	MC	33815	15048	19986	17962	17357	4943	-
	MT	332	619	-	-	-	-	-
Total, period 4		710283	484489	518031	510460	513552	371881	285486
Excess inventory, end of period 4, product:								
	TT	0	0	20813	0	7185	30749	0

Table 5.11 (continued)

(b.)								
Sales forgone: (#)								
Period 1, Product:	KB	0	0	0	0	0	0	33354
	BC	0	0	0	0	0	23765	200599
	MC	0	13458	-	0	0	0	5160
Period 2, Product:	JT	0	0	0	0	0	0	94846
	ST	0	0	0	0	0	94846	0
	KB	0	0	0	0	0	179862	179862
	TT	0	0	0	0	0	0	4909
	BC	0	0	0	163280	135783	212012	212012
	BT	0	0	0	0	1689	0	32150
	MC	0	0	0	0	16595	16595	16595
	MT	0	0	0	0	0	0	2283
Period 3, Product:	KB	0	0	0	0	0	21968	0
	TT	0	0	0	0	0	0	19177
	BC	0	81436	71422	181972	161671	219495	219495
	BT	0	0	0	0	0	0	13400
	MC	0	20130	-	-	20130	0	20130
Period 4, Product:	ST	0	0	0	0	0	0	39219
	SJ	0	1	0	0	0	0	56149
	KB	0	0	50536	0	0	134280	134280
	TT	0	0	0	0	0	0	842
	BC	0	66117	0	32493	26981	198342	198342
	BT	0	0	0	0	0	0	15082
	MC	0	9876	0	4048	5257	30085	30085
Total		0	191018	155546	418968	368106	1131250	1527971

(c.) 10% Increase in demand:

Contribution (1PFT)	$ 283,326	$ 292,461	$ 291,668	$ 281,550	$ 227,580	$ 148,025
Percentage increase in contribution from (b.)	13.5	16.0	25.5	19.5	44.2	16.0
Marginal value of cash	.0575	.05897	1.80884	1.03886	11.91149	10.85437
Implicit funds supplied:						
Period: 1	$ 44,012	$ 41,599	$ 45,435	$ 45,949	$ 31,637	$ 17,166
2	50,655	53,956	40,915	38,147	31,637	16,799
3	33,263	48,595	31,397	10,970	25,351	0
4	23,477	54,853	45,435	8,957	22,447	17,166
Excess inventory, end of period 4, product:						
BC	0	192231	289721	0	0	0
BT	0	12661	0	0	6903	0
MC	0	39223	15160	0	11786	0
MT	0	35708	34512	0	0	0
Total	0	279823	475833	0	18689	0

Table 5.11 (continued)

(c.)							
Sales forgone (#)							
Period 1, Product:	KB	0	0	0	0	0	50528
	BC	0	0	0	0	59919	222393
	MC	0	0	0	0	3882	6891
Period 2, Product:	JT	0	0	0	0	0	61857
	ST	0	0	231	0	74520	104331
	KB	0	0	0	0	197848	197848
	TT	0	0	4286	4286	0	4286
	BC	0	0	0	0	233213	233213
	BT	0	0	5489	25663	0	35365
	MC	0	0	18254	18254	18254	18254
	MT	0	0	0	0	0	2707
Period 3, Product:	TT	0	0	0	0	0	21095
	BC	0	0	0	0	126820	228758
	BT	0	0	0	0	0	1607
	MC	0	0	0	0	0	22143
Period 4, Product:	SJ	0	0	0	0	61764	61764
	KB	0	0	0	0	0	130387
	TT	0	0	0	0	0	926
	BC	0	0	0	0	31347	218176
	MC	0	0	0	0	33093	33093
Total		0	0	28260	48203	770660	1655622

Ideal and Determined Workforce: (manhours)

(d.) Adequate funds:		Optimum Model 09	Planning Model: 01	04	05	02	03	06
Straight time:								
Process, Period								
M1L	1	930	960	-	-	1363	-	1611
	2	1419	1104	-	-	1567	-	1852
	3	1078	1008	-	-	1431	-	1691
	4	942	960	-	-	1363	-	1611
S0L	1	1283	1600	-	-	1891	-	2141
	2	1906	1840	-	-	2174	-	2462
	3	1525	1680	-	-	1985	-	2248
	4	1471	1600	-	-	1891	-	2141
C1L	1	2500	2560	-	-	4082	-	4701
	2	3239	2944	-	-	4695	-	5406
	3	2907	2688	-	-	4286	-	4936
	4	2721	2560	-	-	4082	-	4701
B1L	1	4046	3840	-	-	4800	-	4916
	2	5181	4416	-	-	5520	-	5654
	3	4984	4032	-	-	5040	-	5162
	4	3983	3840	-	-	4800	-	4916
[1]Overtime:								
M1L	1	0	697	-	-	294	-	46
	2	0	161	-	-	0	0	3
	3	6	139	-	-	0	0	0
	4	482	69	-	-	0	0	0

Table 5.11 (continued)

(d.)								
S0L	1	0	643	-	-	353	-	102
	2	118	33	-	-	0	0	0
	3	41	(5)	-	-	0	0	0
	4	496	(31)	-	-	0	0	0
C1L	1	266	2120	-	-	598	598	0
	2	907	985	-	-	0	0	0
	3	599	916	-	-	0	0	0
	4	1463	715	-	-	0	0	0
B1L	1	0	960	-	-	0	0	0
	2	0	843	-	-	0	0	0
	3	0	533	-	-	0	0	0
	4	0	0	0	0	0	0	0
(e.)Minimum funds:								
Straight time:								
Process, Period								
M1L	1	731	960	-	-	984	611	41
	2	1295	1104	-	-	1132	703	47
	3	1078	1008	-	-	1034	642	43
	4	942	960	-	-	984	611	41
S0L	1	1062	1600	-	-	1522	1157	436
	2	1882	1840	-	-	1750	1330	502
	3	1525	1680	-	-	1598	1215	458
	4	1471	1600	-	-	1522	1157	436
C1L	1	2407	2560	-	-	3128	2186	799
	2	3239	2944	-	-	3597	2514	919
	3	2907	2688	-	-	3285	2296	839
	4	2721	2560	-	-	3128	2186	799
B1L	1	1948	3840	-	-	4292	3821	1319
	2	5149	4416	-	-	4936	4394	1516
	3	4984	4032	-	-	4507	4012	1385
	4	3983	3840	-	-	4292	3821	1319
1Overtime:								
M1L	1	0	661	470	(93)	637	781	18
	2	0	13	(333)	(919)	0	0	0
	3	205	(121)	(176)	(767)	(99)	153	0
	4	605	(78)	(14)	(510)	(69)	328	275
S0L	1	0	622	425	(270)	699	830	1
	2	0	(104)	(442)	(1179)	0	0	0
	3	282	(248)	(305)	(950)	(142)	122	0
	4	619	(199)	(137)	(670)	(112)	298	294
C1L	1	0	2037	1552	175	1469	1828	0
	2	643	616	(255)	(1620)	0	0	0
	3	974	268	127	(1329)	(179)	424	0
	4	1710	360	566	(455)	32	917	740
B1L	1	0	652	556	(536)	200	579	0
	2	0	507	0	(576)	0	0	12
	3	462	0	0	(1080)	(387)	0	0
	4	1667	(356)	0	0	0	0	524

Table 5.11 (continued)

(f.) 10% Increase in demand:							
Straight time:							
Process, Period:							
M1L	1	960	-	-	1483	1531	121
	2	1104	-	-	1705	1761	140
	3	1008	-	-	1557	1608	128
	4	960	-	-	1483	1531	121
S0L	1	1600	-	-	2046	2098	536
	2	1840	-	-	2353	2413	616
	3	1680	-	-	2148	2203	563
	4	1600	-	-	2046	2098	536
C1L	1	2560	-	-	4364	4490	949
	2	2944	-	-	5019	5164	1091
	3	2688	-	-	4582	4715	996
	4	2560	-	-	4364	4490	949
B1L	1	3840	-	-	5493	5079	1650
	2	4416	-	-	6317	5840	1898
	3	4032	-	-	5768	5333	1733
	4	3840	-	-	5493	5079	1650
[1]Overtime:							
M1L	1	948	872	0	425	341	0
	2	343	268	(594)	0	0	(15)
	3	304	304	(53)	0	0	174
	4	181	181	0	0	(19)	382
S0L	1	946	880	(112)	500	422	0
	2	255	190	(797)	0	0	(26)
	3	195	195	(186)	0	0	240
	4	89	89	0	0	0	441
C1L	1	2720	2563	403	916	701	0
	2	1426	1269	(876)	0	0	0
	3	1320	1320	396	0	0	444
	4	1001	1001	701	0	0	1045
B1L	1	1653	766	0	0	0	0
	2	1442	555	(10)	0	0	135
	3	1058	1059	0	0	0	1053
	4	276	276	0	0	0	1119

1, Figures in parentheses are idle manhours or unused straight time.

For this reason, in the first step of the test procedure the models perform only a forecasting or budgeting function.

The funds estimates shown in Table 5.11(a) reflect the total amount of implicit financing needed in each period if the corresponding optimal production plan is adopted. A comparison of these production schedules shows that there are very few significant differences between them. If the corresponding plan developed by the more lightly constrained Optimum Model 09 is used as a bench mark, additional comparisons do point out that the planning models require higher levels of output in the earlier periods. This shift in production is the expected result of the minimum intra-cycle inventory requirement. In turn, higher inventory levels are reflected in financing requirements and all planning models need more funds in every period than Optimum Model 09.

More financing will also be needed if the model "determined" workforce is employed rather than management's "ideal" workforce. This is particularly true in period four and does not seem consistent with the reduction in use of overtime associated with the determined workforce (see section d). To see why more funds are required it is necessary to check inventory on hand at the end of the planning horizon. All models have some excess inventory but 02, 03 and 06 have significant amounts. This result follows from the assumption implicit in functional (10A-2) that both straight time hours and inventory maintenance have no direct cost. For this reason inventory accumulation allows the model to determine a production schedule that minimizes both overtime and idle time. The largest amounts of excess inventory are accumulated by 06 because relative profitability among the products has changed with the lower marginal value of cash.

Comparing short-term financing alternatives indicates that refinancing overdue current liabilities has a minimal impact on profit. The significant jump in contribution between 03 – 06 and 04 – 05 reflects about $10,000 in interest saved by liquidating the beginning factor balance. If term financing can be arranged at less than the marginal cost of funds some of this saving will be realized. However, contrasting 04 and 05 with 01 does show that elimination of payments on overdue accounts starts to

reduce the need for outside financing almost immediately. In evaluating cash requirements in this case it should be noted that interest payments (if any) on the hypothetical term loan are not included in cash outflows (Table 5.9) and that period four is on the down-side of a seasonal peak and funds are flowing out of receivables.

Differences in profit among the plans are generally small after adjustment for the interest saving mentioned above. However, in every instance the determined workforce proposal is about $5,000 more profitable than the ideal workforce and it is no less profitable than Optimum Model 09. This observation implies that if financing can be arranged, minimum intra-cycle inventory and stable workforce proposals can be implemented without adversely affecting profit. This is particularly true the more nearly optimal the size of the workforce that can be employed. However, in evaluating profit response to various proposals it does not necessarily follow that proportionate changes in indirect costs will also occur and in most cases the absolute increase (or decrease) in contribution will flow through intact to net profit. In COMPA, considering the profit levels shown in Appendix A to Chapter 4, a few thousand dollars more or less would change net operating results substantially.

The manhours distributions shown in Table 5.11 (d) are employment plans directly related to the production schedules in (a). When the ideal workforce is used these schedules are identical, regardless of the financing assumption, and straight time hours utilized are also quite similar to Optimum Model 09. The determined workforce is larger in all periods and processes because the model is attempting to eliminate both overtime and idle time. To the extent that excess inventory can be used in a later period, the determined workforces are acceptable, but there are other considerations that cast doubt on their optimality outside the model. First, the July to September quarter is a seasonal high and a realistic plan should specify excess beginning rather than ending inventory. This shortcoming should disappear as the planning horizon is extended. Second, the ideal and determined workforces are not directly comparable because management's estimates are on an annual rather than a

four-month basis and they also reflect a willingness to accept overtime in preference to idle time.

The second step in the planning test uses the sequential parametric routine described earlier in this chapter to minimize cash requirements in every period. Because this technique minimizes the period of highest demand first it also has a leveling effect. That is, requirements increase in periods of lesser demand to offset funds denied in the period of maximum demand. Comparing fund requirements and profit in Table 5.11(b) with comparable quantities in (a) shows immediately that funds are needed when they are needed. Reallocating production to minimize cash needs in the period of highest demand is surprisingly expensive and ineffective. The extremely high marginal values determined for cash are only valid within very small ranges but they serve to underscore how valuable a dollar can be at the point of insolvency.

Possibly the most interesting contrast provided by the analysis is the relative ability of the models to minimize funds and maintain feasibility (stay in business). 01 and 02 display very limited flexibility and 01's average fund requirements actually increase from $51,000 to $55,000 because adequate funds are withheld in period 2. These models are more rigid than 03 through 06 because they have larger fixed cash outflows and because factoring relates supply of funds to sales. With this type of financing there is a direct link between production, sales and availability of funds and neither 01 nor 02 can conserve cash by forgoing a significant amount of available demand. In 03 and 04 lower fixed outflows and functional relation of more flows to decision variables provide more flexibility. However, it is again noted that in 04 average demand for funds increases, in this instance from $36,000 to $46,000. This result can be explained by comparing idle time in (e) from models 01 and 04 with 02 and 03. Because the stipulated ideal workforce cannot be reduced, rescheduling and cutting back production creates a considerable labor surplus. 02 and 03 are better able to cope with a reduced supply of funds because they have control of their labor inputs.

The most dramatic responses to funds rationing were made by 05 and 06. Although fixed outflows are the same as specified in 03 and 04, separating financing from sales by eliminating the factor as a source of funds clearly increases the options available to the firm. In both models significant portions of total demand are rejected with equally significant impacts on contribution. In this respect, 05 and 06 both react to funds rationing in a "traditional" way. That is, they retrench and make profit sacrifices that are disproportionate to the reductions in demand for funds that are achieved.

The next phase of the planning evaluation investigates response to a 10% increase in demand, assuming that the firm is operating under the absolute funds rationing imposed in phase two. To simulate this situation the right-hand side values for rows 1-4D_j and 4F_j were parametrically increased by 10%. Comparing contribution and marginal values of cash in Table 5.11 (c) with those in (b) indicates that in all cases the models were able to profitably use additional external opportunities. How advantageously increases in demand were exploited is inversely related to each model's ability to conserve cash. This primarily reflects the advantage of starting from a higher base but models 01 and 02 also clearly point out how factoring can convert incremental demand into instant cash. The same observation is true for 03 and 04 although a 10% increase in demand did not permit these models to generate all the cash that could be used.

The noticeably better response of 05 to incremental demand relative to 06 is again attributable to the workforce assumption. Because additional demand permitted both an increase in total sales and a reallocation of resources, idle time could be reduced with substantial benefits to profit. 06 did not have surplus labor available and workers had to be added. In addition, 06 is still required to forgo a considerable portion of total demand. The ability of linear programming to identify the most profitable marginal opportunities is apparent if sales forgone by 06 in (c) are compared with (b). For example, in (b) demand for 33,354 pounds of 1DKB was rejected. When external opportunities were

increased by 10%, this quantity increased to 50,528 pounds, or 50%. The model continued to reduce sales of this product in order to accept additional sales of more profitable items.

The planning tests summarized in Table 5.11 do not constitute a rigorous objective proof of the usefulness of linear programming for short-term financial planning. Rather these tests provide evidence and a finding based on this type evidence is necessarily subjective and must be qualified. That is, the planning tests cannot establish that the technique is useful, only that it may be useful. However, this evidence does show that the fully integrated linear programming model can provide meaningful insights into the similarities, differences and relative desirability of complex proposals.

Linear programming production models have been successfully used for sub-optimal planning in many actual situations. Although the fully integrated aggregate model provides a less detailed production schedule, it includes the financial and policy aspects that must be considered if returns to the firm are to be optimized. The tests that have been made using models 01 through 06 point out that inclusion of these financial and policy relationships and restrictions markedly effects the optimal plans that are developed. The amount of funds available, the terms under which they are supplied, the periods of availability and their explicit cost all must be reflected in optimal production schedules, inventory policies, and employment practices. Conversely, all operating plans and policies have financial impacts that must be recognized when changes are considered, budgets prepared, etc. This feedback between operating, financial and policy decisions is clearly apparent in the planning models.

There is normally no need to ration funds within the firm if an adequate external supply is available. Under these circumstances, in addition to evaluating the financial implications of operating plans, the fully integrated model can be extended to determine the least cost "financing package." Robichek, Teichroew, and Jones[14] have shown that linear programming can optimize the short-term

[14] *op. cit.*

financing decision, i.e., minimize total interest cost. However, their approach is financial sub-optimization because sources and uses of funds are not optimized jointly. A fully integrated model that recognizes financing options would not only minimize financing costs but also insure that the marginal value of cash does not fall below the explicit cost of borrowing.[15]

When funds are rationed, as in phases two and three, the power of linear programming to allocate limited resources to their most profitable opportunities is clearly apparent. The fully integrated model can define the best possible response to limited funds because decisions are based on marginal analysis and it determines the optimal level of operations consistent with identifiable sources of financing. If a firm faces a steeply rising funds supply schedule the fully integrated model can also identify the point at which the firm should stop borrowing.

5.5 Evaluation of the Derived Marginal Revenue Product Functions

It was shown in a previous section that the fully integrated model makes it possible to identify the general shape of the marginal revenue product curve for cash through the use of parametric programming. Although this type of analysis is mathematically precise, results must be interpreted with caution. Experimentation has shown that in all cases the COMPA fully integrated models can be deprived of funds until their solutions are forced into infeasibility. This result follows because these models contain rigidities in the form of fixed minimum fund requirements and at the point of infeasibility all options, regardless of cost, have been exhausted. In addition, as the COMPA models approach infeasibility they become less representative of the firm because under actual conditions of adversity management can reduce mandatory cash payments by assuming other risks. The models do not contain discretionary options of this sort and the loss of representativeness causes the extremely high shadow prices recorded at the point of infeasibility. In the flatter portions of the curve the

[15] To implement this concept, the R, T, and J financing matrix would be joined to the fully integrated model and the constant R_j in their requirements equation (*ibid*, p. 13) would be replaced by the implicit funds variable, e.g., LB$ in models 05 and 06.

shadow prices that are predicted appear realistic. However, even in this range the model uses the last dollar optimally and assumes that the next one will be employed in the same manner. In COMPA one dollar more or less would probably never be noticed.

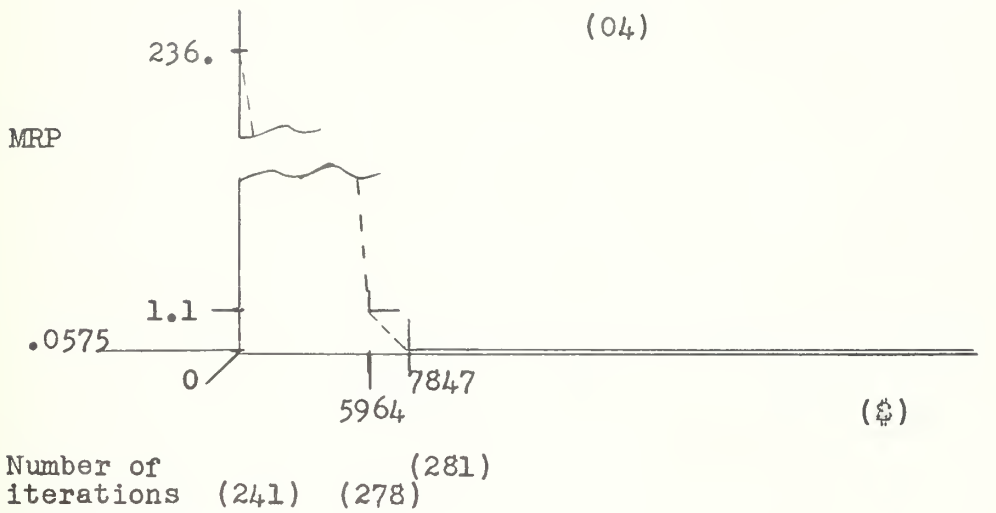
The marginal revenue product curves for models 03 through 06 have been traced by parametrically increasing the amount of funds available and are shown in Figures 5.3 and 5.4. Unfortunately, the practical limitations of the LP-90 parametric routine do not provide enough observations to give more than a general indication of their shapes. Comparing the curves from the ideal workforce models 04 and 05 with those obtained from models 03 and 06 again indicates how a reduction in the level of constraint provides more opportunities for cash conservation and extends the range within which the marginal value of cash exceeds its explicit cost. This same result is apparent when the factoring arrangement is eliminated and the supply of funds is no longer contingent on sales.

The specific shapes of the marginal revenue product curves shown in Figures 5.2, 5.3, and 5.4 are directly determined by the characteristics of the COMPA models. These curves naturally reflect diminishing marginal product but the marginal value at the point of minimum resources and the point on the horizontal axis at which marginal product falls to zero (or its minimum value) are traceable to the assumptions incorporated in the model being tested. As noted in regard to COMPA, both the rigidities imposed by fixed cash flow requirements and the number and range of opportunities have an impact on the slope of the marginal product function.

The lightly constrained models used in theoretical examples generally are shown to define a marginal revenue product curve for funds that has a gentle negative slope over a wide range. Typically these models are only subject to internal constraints, i.e. capacity, resources and perhaps funds. Although the range of the external opportunity set is also defined, specified opportunities can be exploited without limit. This situation provides a shadow price for cash that results from the model trading off among external opportunities in order to maximize utilization of limited inputs. However,

Figure 5.3
COMPA

Planning Models 04 and 05
Marginal Revenue Product, XCA



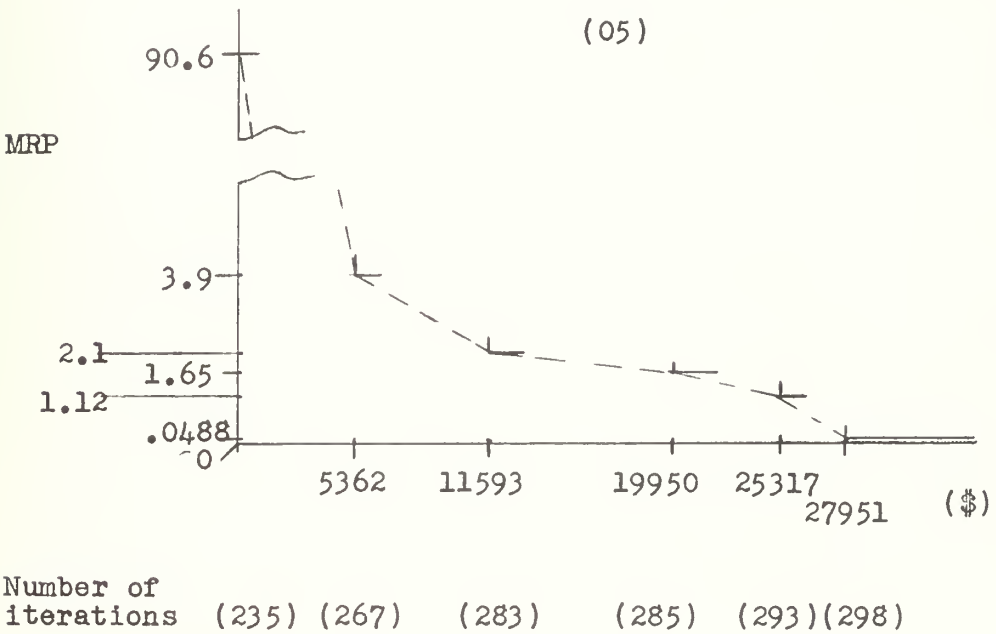
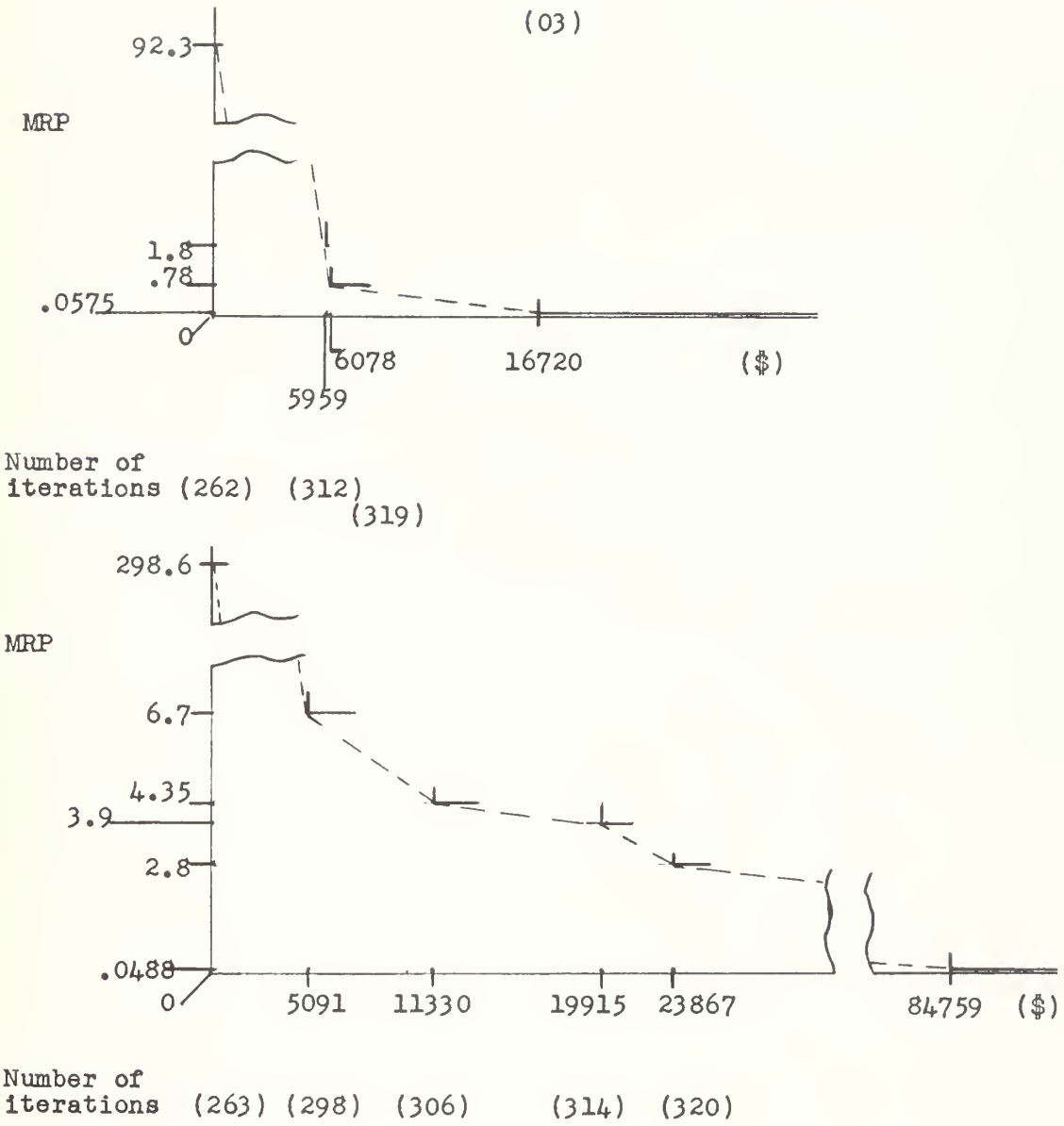


Figure 5.4
COMPA

Planning Models 03 and 06
Marginal Revenue Product, XCA



if external opportunities are also limited, the shadow price of cash reflects marginal trade-offs made among external and internal limitations.

If management understands its limitations, the marginal value of funds determined by a fully integrated model can provide valuable, otherwise unobtainable, information for use in financial decision making. However, this information must be thoroughly understood and carefully used because it is derived from a model and it can only apply to decisions that involve one or more of the decision variables included in the model. In addition, model values cannot be extrapolated outside the range within which the model has been validated.

The parametric method used in this paper to trace the marginal revenue product curves is not considered to be either a necessary or practical approach to the operational use of the funds shadow price. A more useful approach for a real firm would be to specify the expected level of actual financing and determine if a feasible solution can be obtained. If a feasible solution is possible and funds are binding the supply can be increased from the level stipulated and a meaningful schedule obtained. If in the initial step a feasible solution is not possible, opportunities to conserve cash within the model (and the firm) should be investigated before an implicit source of funds is used.

In some cases it may be difficult for the financial manager to adequately understand the shadow price analysis. However, a somewhat indirect approach can be equally useful. If a fully integrated model is available and used regularly for planning analyses the shadow price does not have to be considered directly. Rather the model should be formulated to also determine a "financing package" and he can then be sure that linear programming has used the shadow price information in an optimal manner. This tactic will not avoid the problem if the firm is subject to absolute funds rationing. However, very few firms appear to be completely cut off from all sources of additional short-term funds at non-infinite cost.

5.6 The Ten Period Planning Models

The COMPA models used thus far in the study have been limited to four periods to facilitate discussion and understanding. However, in the case of COMPA, a four-month horizon is not adequate for short-term planning and quarterly periods are too insensitive. In this section the models will include ten periods (months) to demonstrate that the horizon can be readily extended and that plans produced by these more comprehensive models are quite realistic. In fact, if they were based on projections rather than historical data, the plans that are developed in this section would be immediately useful to COMPA. Although ten periods may appear to be an unusual planning cycle, the models are intended to represent a six-month horizon plus four terminal periods. The terminal periods are included primarily to mitigate the finite horizon problem created when ending conditions are specified.

To accomplish the specific objectives of this section it is not necessary to repeat all the tests performed with the four period models. For this reason only models 05 and 06 will be extended to represent the ten month period, January through October 1966. This extension could be accomplished by modifying the parameter "M" and adding the necessary right-hand side values. However, to simplify the analysis the coefficient q_T in equation (11-2) will be set equal to unity in all periods, $q_T = 1$. This change incorporates the assumption that each month has an equal number of work days. Equation (1) will also be changed to the non-cumulative form for models 010 and 011. This is a technical programming change to reduce the density of the matrix and conserve computer time. When revised, equation (1) has the form:

$$B_{jT-1} + P_{jT} - S_{jT} = B_{jT}$$

$$-P_{jT} + S_{jT} - B_{jT-1} + B_T = 0 \qquad (1\text{-}1)$$

$$T = 1,M$$

$$j = 1,N$$

The additional right-hand side values required for the ten period models are summarized in Appendix E. Because the months of July through October are included in the analysis most of the right-hand side developed for models 05 and 06 (Appendix D) can be used and July becomes period 7 and so on. Calculation of the revised right-hand side for equation (8-3) is identical to that shown in Table 5.9 and it will not be included.

The basic proposals that will be evaluated using planning models 07 through 011 are essentially those carried forward from 05 to 06. In all cases it is assumed that overdue accounts and the initial factor balance were refinanced prior to the planning cycle. This permits non-model related cash flows to be estimated on the basis that only current payments are required. No initial overdraft is specified because on January first, year-to-date cash payments cannot exceed receipts. Other assumptions reflected in the models are summarized in Table 5.12.

Table 5.12
COMPA

Planning Models 07-011
Summary of Planning Assumptions

		Long Term Financing: Term Loan			
	07	08	09	010	011
Minimum Inventory:					
Periods 1–9		x	x	x	x
Period 10	x	x	x	x	x
Workforce:					
"Ideal"			x	x	
Determined by Model					x
Short Term Financing:					
Line of Credit	x	x	x	x	x

Model 07 is very lightly constrained and is included to show the best performance possible during the planning cycle. Equation (11-2) has been removed from 07 to give the model unlimited flexibility

to schedule production to known sales and only ending inventory levels are specified. An ample supply of straight time labor is provided and equation (13) is used to permit the model to vary straight time manhours to meet production demands.

Straight time labor utilization is also unconstrained in model 08. However, to show the impact of minimum intra-cycle inventories on both fund requirements and workforce flexibility, equation (11-2) has been included in this model.

Management's ideal workforce is specified in model 09. Each month is assumed to have 21 work days or 168 productive hours per employee and the quantity of straight time allowed each process is shown in Appendix E. To provide an additional point of comparison a cost penalty for idle time is not employed. This variation permits 09 to minimize overtime without regard to the presence of unused hours in other periods.

Models 010 and 011 are direct extensions of 05 and 06. Both incorporate the minimum inventory equations (11-2) and (11-2A) and idle time penalties are operational. The ideal workforce is specified in 010 and 011 is constrained to determine an optimal size workforce.

5.7 Evaluation of the Ten Period Planning Models

The test procedure to be followed in this section is straight-forward and intended primarily to show how the ten period models can be used. Sequentially more constrained problems are solved to demonstrate that realistic schedules, plans, requirements, etc., can be defined. The plans indicate an optimal response to operating conditions and the profit and cash flow effects of more restrictive policies are identified. In a second phase, the impact of a 10% increase in demand on 010 and 011 is evaluated.

The ten period aggregate models are approximately the same as the comprehensive model, i.e., 450 rows by 500 columns, excluding slack vectors. The ability to extend the horizon by six periods without increasing the matrix points out how the usability of the COMPA fully integrated model

has been enhanced by aggregation of products. For example, the number of rows in 010 are defined by the expression, 3MN + 17M + 1. The impact that additional periods (M) or products (N) have on model dimensions is clear. However, with N = 10, as in the COMPA aggregate models, a planning horizon of up to 18 periods can be accommodated by LP-90. The substitution of equation (1-1) for (1) reduced matrix density from 1.14% to .99% and may have saved a small amount of running time.

Solutions obtained from models 07 through 011 are summarized in Table 5.13. Production plans are not shown but excess inventory at the end of each period is used to indicate how the several models schedule production during the planning cycle. Labor inputs have been converted from manhours to number of workers. In 010 and 011 the number of workers in each process is constant from period to period because of the stable workforce constraints. In 07 and 08 the model has complete control over the supply of straight time labor and the range and periods of maximum and minimum demand are shown for these models. External fund requirements in each period are also shown and figures in parentheses represent cash on hand. The marginal values of cash generally reflect the explicit interest rate compounded monthly. In 07 through 011 I' is set at .01 or 12% per annum. If there is excess cash on hand at the end of any period the shadow price is naturally zero and this is reflected in a lower value for the cumulant.

The contribution estimates shown in Table 5.13 are difficult to evaluate in absolute terms. A comparable actual contribution of $610,350 was calculated by the procedure used in Table 4.12. However, factor interest charges are included and it is difficult to determine how much of this amount to add back. If the January 1, 1966, factor balance of $180,000 had been refinanced about $25,000 in interest would not have been paid to the factor during the next ten months. Incorporating this adjustment provides a comparable actual contribution during the planning cycle of about $635,000. All the models generate somewhat more contribution but not significantly larger amounts. In this regard 07 can be assumed to indicate the most contribution that could have been realized if management had had complete foreknowledge of demand and infinite control of the labor force.

Table 5.13
COMPA

Comparison of Plans and Estimates Developed by
Planning Models 07 through 011, 010-1 and 011-1

	07	08	09	010	011	010-1	011-1
Contribution (1PFT)	$ 654,835	$ 652,725	$ 644,908	$ 641,604	$ 649,177	$ 690,612	$ 704,970
Marginal value of cash	0	.09368527	.09368527	.104622	.104622	.104622	.200221
Implicit funds supplied:							
Period: 1	$ (9,204)	$ 4,875	$ 7,701	$ 12,432	$ 14,738	$ 14,129	$ 16,713
2	(14,317)	15,064	24,364	37,056	46,293	40,006	50,234
3	(2,249)	19,755	35,526	51,275	67,627	57,106	72,052
4	0	39,351	58,644	74,596	95,179	81,129	100,000
5	(15,181)	44,111	61,983	78,955	96,473	78,633	100,000
6	(21,527)	32,330	46,839	63,959	77,625	56,060	79,312
7	(9,965)	32,610	46,652	62,618	73,090	55,958	74,897
8	0	21,482	31,896	45,811	51,381	48,324	52,337
9	(13,725)	1,373	8,797	21,144	21,491	30,503	21,936
10	0	(16,438)	(8,493)	13,597	7,789	31,760	18,391
Normal workforce:	Max. Min.	Max. Min.					
Process: M1L	10.9(8) 2.3(1)	7.8(8) 4.5(2)	6	6	6.6	6	7.5
S0L	15.1(8) 4.5(4)	11.3(8) 7.2(2)	10	10	9.6	10	10.5
C1L	31.0(8)10.2(1)	23.6(8)15.5(2)	16	16	20.4	16	24.2
B1L	49.7(9)12.8(4)	31.6(8)17.1(2)	24	24	24.5	24	27.4
Excess F.G. inventory: (#)							
End of period:	(Note 1)						
1	50500	1484	19693	59053	103411	55008	89941
2	58632	0	41102	130751	219383	123360	202714
3	157292	0	62371	153017	284955	158728	261529
4	1360	0	77513	165302	313520	153137	273432
5	0	0	47473	131760	260746	63761	198169
6	14505	0	32883	114964	225491	0	148621
7	105378	0	28221	97114	190869	0	99840
8	278645	0	1441	54770	109660	0	23834
9	214467	0	0	56160	86868	0	4648
10	106092	0	0	0	1236	0	0

Note 1:
In 07 only period 10 is excess inventory. Amounts shown for other periods are total finished goods inventory on hand at the end of the designated period. Intra-cycle minimums were not required in this model.

Moving from left to right in Table 5.13 shows that as the models are required to operate with more restrictions, profitability is reduced and cash requirements increase. This is certainly the response that would have been predicted but not all results are intuitively obvious. For example, comparing 010 with 07 indicates that adopting minimum intra-cycle inventories and a stable workforce reduces profit and increases external fund needs. However, 08 and 09 show that over one-half of the additional funds are required to implement only the minimum inventory policy and that profit effects are attributable almost entirely to the overtime associated with the stable workforce. Proceeding to 011 it is also noted that a more nearly optimal workforce restores most of the lost contribution but continues to increase fund requirements.

The workforce section of Table 5.13 shows that 07 requires severe dislocations in the workforce between the periods of maximum and minimum demand. This effect is mitigated considerably by the minimum intra-cycle inventory requirement and the ideal workforce specified by management is within this narrower range. The optimality of the ideal workforce was questioned when the four period models were used but the more realistic ten period horizon verifies that it is close to the best possible workforce as determined by 011.

The excess inventory patterns are also of considerable interest. Intuitively 07 should operate with no intra-cycle finished goods inventory and certainly no excess ending inventory but it is shown to accumulate both. The inventory policy required to implement stable employment is also clear. When stable employment is assumed there is a need to build inventory in the first four months in anticipation of the peak season in periods 5 through 9. However, the model has scheduled production so that no excess inventory is carried over into the very slow November–December period. The accumulation of inventory is naturally reflected in the amount of financing required in each period.

The second phase of the test investigates the impact of a 10% increase in demand on models 010 and 011. Assuming that a maximum line of credit of $100,000 is available, demand for each product in

every period is increased by 10%. Required ending inventories and mandatory cash outflows are also increased proportionately. 010 is able to absorb these changes within available funds because the fixed workforce assumption requires that mostly overtime be used to increase production. The model will not use overtime to produce for inventory and funds are released. However, these funds are required for overtime payroll and although contribution increases significantly total fund requirements are essentially the same as 09.

Model 011 uses the added demand more profitably because it can add to its stable workforce. The extra workers also permit an orderly pattern of inventory accumulation to be maintained. However, this takes more funds and the line of credit is fully utilized in periods 4 and 5. Because more funds could be profitably used the shadow price of cash has increased from .105, the explicit rate, to .20. This higher shadow price reflects wholly internal opportunities for production economies because all external opportunities (demand) have been accepted. It is also interesting to note how conservatively 011 increased the basic workforce. Although small amounts of overtime were used much of the increased demand was absorbed by reducing inventory accumulation and there are probably opportunities for more overtime/inventory trade-offs if additional funds could be provided.

Marginal revenue product curves for cash were not traced for the ten period models. Those developed from the four period models demonstrate that they can be defined. Repeating that phase of the test here would provide no additional insights and it is not considered that defining curves for models 07 through 011 would justify the computer time required.

The ten period models clearly show that the power of linear programming can be focused on realistic financial planning problems. Although in these tests the fully integrated model was generally not required to ration funds the capability is always present and becomes operational when needed, as in 011-1. Even in the absence of funds rationing the fully integrated model provides a planning vechicle that can be used to analyze large and complex interrelated financial/operational problems rapidly and in depth. Through the combination of linear programming and modern data processing capabilities,

the quantity and quality of financial planning that becomes practical is impressive. When a fully integrated model is available management can consider all practical courses of action and make decisions with the knowledge that each has been completely and consistently analyzed. With this information in hand businessmen should know the implications of a decision on all important facets of their firm. Management cannot delegate decisions to linear programming but the fully integrated model can provide better financial information for decision making.

Chapter 5
Appendix A

Optimum Model 02
Optimal Primal-Dual Solution

1PFT 267342.81

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0 00000	267342.80962994	1PFT	1.00000000	.
0 00000	288465.03746894	2NSP	.	.
0 00000	46633.92810324-	2PFT	.	.
0 00000	.	2DLC	.	.
0 00000	.	3DLC	.	.
0 00000	.	4DLC	.	.
1PJT	40439.00000000	1GPF	1.00000000-	6785.60000000-
1PST	20431.00000000	2GPF	1.00000000-	6785.60000000-
1PSJ	.	3GPF	1.00000000-	6785.60000000-
1PKB	129042.00000000	4GPF	1.00000000-	6785.60000000-
1PKT	1241.00000000	1SPN	.	8990.00000000-
1PBC	200599.00000000	2SPN	.	8990.00000000-
1PBT	55977.43641660	3SPN	.	8990.00000000-
1PMC	5160.00000000	4SPN	.	8990.00000000-
1PMT	3144.00000000	1M1L	.10380000	485.70000000
2PJT	97539.00000000	1S0L	.10380000	346.90000000
2PST	94846.00000000	1C1L	.98740000	471.90000000
2PSJ	.	1B1L	.09300000	625.50000000
2PKB	189054.47778106	2M1L	.07740000	557.70000000
2PKT	4405.00000000	2S0L	.77012451	400.90000000
2PTT	4909.00000000	2C1L	.94620000	552.90000000
2PBC	212012.00000000	2B1L	.13516278	691.50000000
2PBT	32615.20711624	3M1L	.87640000	509.70000000
2PMC	16595.00000000	3S0L	.87640000	364.90000000
3PJT	83964.00000000	3C1L	.90500000	498.90000000
3PST	85063.00000000	3B1L	.15460017	647.50000000
3PSJ	.	4M1L	.83680000	485.70000000
3PKB	162122.52221894	4S0L	.83680000	346.90000000
3PKT	4164.00000000	4C1L	.86380000	471.90000000
3PTT	25155.00000000	4B1L	.30586263	625.50000000
3PBC	219495.00000000	1TJT	.00017291	27276.00000000
3PBT	18806.35646716	1TST	.00131600	35087.00000000
3PMC	20158.37007710	1TSJ	.00154975	.
4PJT	70571.00000000	1TKB	.00029233	35856.00000000
4PST	56624.00000000	1TKT	.00303248	2731.00000000
4PSJ	57531.00000000	1TTT	.	3182.00000000
4PKB	206675.00000000	1TBC	.00092791	17340.00000000
4PKT	2732.00000000	1TBT	.	1238.00000000
4PBC	245028.00000000	1TMC	.00131838	12155.00000000
4PMC	33786.62992290	1TMT	.	1063.00000000
4PMT	332.00000000	2TJT	.00073157	27276.00000000
1SJT	67715.00000000	2TST	.00175920	35087.00000000
1SST	55518.00000000	2TSJ	.00199724	.

1SSJ	.	2TKB	.	35856.00000000
1SKB	164898.00000000	2TKT	.00317989	2731.00000000
1SKT	3972.00000000	2TTT	.00331693	3182.00000000
1STT	1591.00000000	2TBC	.00019607	17340.00000000
1SBC	217939.00000000	2TBT	.	1238.00000000
1SBT	13531.00000000	2TMC	.00053790	12155.00000000
1SMC	17315.00000000	2TMT	.	1063.00000000
1SMT	974.00000000	3TJT	.00210329	27276.00000000
2SJT	97539.00000000	3TST	.00262397	35087.00000000
2SST	94846.00000000	3TSJ	.00283711	.
2SSJ	.	3TKB	.00252193	35856.00000000
2SKB	179862.00000000	3TKT	.00315100	2731.00000000
2SKT	4405.00000000	3TTT	.	3182.00000000
2STT	6500.00000000	3TBC	.00278231	17340.00000000
2SBC	212012.00000000	3TBT	.	1238.00000000
2SBT	32150.00000000	3TMC	.	12155.00000000
2SMC	16595.00000000	3TMT	.00203063	1063.00000000
2SMT	2461.00000000	4TJT	.01679360	27276.00000000
3SJT	83964.00000000	4TST	.01584914	35087.00000000
3SST	85063.00000000	4TSJ	.01590695	.
3SSJ	.	4TKB	.01479291	35856.00000000
3SKB	171315.00000000	4TKT	.02071135	2731.00000000
3SKT	4164.00000000	4TTT	.01513453	3182.00000000
3STT	19177.00000000	4TBC	.01371164	17340.00000000
3SBC	219495.00000000	4TBT	.02639836	1238.00000000
3SBT	31959.00000000	4TMC	.01695125	12155.00000000
3SMC	20130.00000000	4TMT	.01983489	1063.00000000
3SMT	772.00000000	1SUG	.08000000	687.00000000-
4SJT	58402.00000000	2SUG	.06000000	687.00000000-
4SST	39219.00000000	3SUG	.04000000	687.00000000-
4SSJ	56149.00000000	4SUG	.02000000	687.00000000-
4SKB	134280.00000000	1ORM	.06000000	1064.00000000
4SKT	2086.00000000	2ORM	.04000000	1064.00000000
4STT	842.00000000	3ORM	.02000000	1064.00000000
4SBC	198342.00000000	4ORM	.	1064.00000000
4SBT	15082.00000000	1XWJ	.04240000	.
4SMC	30085.00000000	2XWJ	.04160000	.
4SMT	92.00000000	3XWJ	.04080000	.
1BTT	1591.00000000	4XWJ	.04000000	.
1BBT	43684.43641660	1XWC	.03180000	.
1BMT	3233.00000000	2XWC	.03120000	.
2BKB	9192.47778106	3XWC	.03060000	.
2BBT	44149.64353284	4XWC	.03000000	.
2BMT	772.00000000	1XSJ	.06370452-	966.00000000-
3BTT	5978.00000000	2XSJ	.04558804-	966.00000000-
3BBT	30997.00000000	3XSJ	.02790192-	966.00000000-
3BMC	28.37007710	4XSJ	.01224224-	966.00000000-
4BJT	12169.00000000	1XSC	.05548169-	8040.00000000-

4BST	17405.00000000		2XSC	.03810737-	8040.00000000-
4BSJ	1382.00000000		3XSC	.02333856-	8040.00000000-
4BKB	72395.00000000		4XSC	.00950596-	8040.00000000-
4BKT	646.00000000		1XFB	.02000000-	175100.00000000
4BTT	5136.00000000		2XFB	.02000000-	175100.00000000
4BBC	46686.00000000		3XFB	.02000000-	175100.00000000
4BBT	15915.00000000		4XFB	.02000000-	175100.00000000
4BMC	3730.00000000	-	1XFM	.00515000-	89550.00000000-
4BMT	240.00000000	-	2XFM	.00542500-	39800.00000000-
1RTM	915.25511361	-	3XFM	.00544000-	.
2RTM	1317.39223342	-	4XFM	.00572000-	.
3RTM	1078.00000000	+	1XLC	.	100000.00000000
4RTM	942.00000000	+	2XLC	.	100000.00000000
1RTS	1252.96621780	+	3XLC	.	100000.00000000
2RTS	1906.00000000	+	4XLC	.	100000.00000000
3RTS	1525.00000000		1XCA	.02000000	49793.00000000-
4RTS	1471.00000000		2XCA	.02000000	85166.00000000-
1RTC	2500.00000000		3XCA	.02000000	109931.00000000-
2RTC	3239.00000000		4XCA	.02000000	147909.00000000-
3RTC	2907.00000000		4FJT	.01679360-	12169.00000000
4RTC	2721.00000000		4FST	.01584914-	17405.00000000
1RTB	4046.17166365		4FSJ	.01590695-	1382.00000000
2RTB	5181.00000000		4FKB	.01479291-	72395.00000000
3RTB	4984.00000000		4FKT	.02071135-	646.00000000
4RTB	3983.00000000		4FTT	.01513453-	5136.00000000
3ETM	121.89106613		4FBC	.01371164-	46686.00000000
4ETM	481.70089896		4FBT	.02639836-	15915.00000000
3ETS	189.77577848		4FMC	.01695125-	3730.00000000
4ETS	496.39710859		4FMT	.01983489-	240.00000000
1ETC	226.43709352	+	1DJT	.19306490	67715.00000000
2ETC	690.82529435	+	1DST	.10132219	55518.00000000
3ETC	855.06112871	+	1DSJ	.04022406	.
4ETC	1462.98418140	+	1DKB	.11393793	164898.00000000
1GUS	37702.99834042	+	1DKT	.19851162	3972.00000000
2GUS	53970.88068117	+	1DTT	.05150131	1591.00000000
3GUS	49362.37186653	+	1DBC	.08137648	217939.00000000
4GUS	54935.45983964	+	1DBT	.14105004	13531.00000000
1RRM	29982.07269990	+	1DMC	.03080611	17315.00000000
2RRM	45243.10704895	+	1DMT	.15740886	974.00000000
3RRM	44386.58416457	+	2DJT	.18568085	97539.00000000
4RRM	45212.48190325	+	2DST	.09624885	94846.00000000
1WGJ	143301.00000000	+	2DSJ	.03614246	.
2WGJ	237901.00000000	+	2DKB	.10923573	179862.00000000
3WGJ	225099.00000000	+	2DKT	.18821415	4405.00000000
4WGJ	171872.00000000	+	2DTT	.04571706	6500.00000000
1WGC	400152.00000000	+	2DBC	.07819317	212012.00000000
2WGC	408469.00000000	+	2DBT	.13284978	32150.00000000
3WGC	410940.00000000	+	2DMC	.02835485	16595.00000000

	4WGC	362707.00000000	+	2DMT	.15001568	2461.00000000
	1SJ$	65072.90221039	+	3DJT	.17905399	83964.00000000
	2SJ$	106481.59895045	+	3DST	.09178949	85063.00000000
	3SJ$	99897.92336018	+	3DSJ	.03266113	.
	4SJ$	72870.15832090	+	3DK8	.10508018	171315.00000000
	1SC$	124588.21773869	+	3DKT	.17839975	4164.00000000
	2SC$	127618.38041844	+	3DTT	.04340615	19177.00000000
	3SC$	127784.38919864	+	3D8C	.07498453	219495.00000000
	4SC$	112626.95421922	+	3DBT	.12486331	31959.00000000
	1FP$	143758.47355447	+	3DMC	.02577840	20130.00000000
	2FP$	169196.06956286	+	3DMT	.14281713	772.00000000
	3FP$	181197.54024757	+	4DJT	.17473363	58402.00000000
	4FP$	165016.40891875	+	4DST	.08899907	39219.00000000
	1F8$	182709.97989874	+	4DSJ	.03073905	56149.00000000
	2FB$	199600.17242689	+	4DKB	.10374801	134280.00000000
	3F8$	199567.13007095	+	4DKT	.17013698	2086.00000000
	4F8$	182556.17412147	+	4DTT	.03851479	842.00000000
	1L8$	19606.13046167	+	4D8C	.07461599	198342.00000000
	2L8$	12107.38035063	+	4D8T	.11788358	15082.00000000
	3LB$	2405.77957274	+	4DMC	.02289950	30085.00000000
	4L8$	13466.35053612	+	4DMT	.13856567	92.00000000
	1GP$	83856.89374689	+	1XTM	.	930.00000000
	2GP$	101679.80712509	+	2XTM	.	1419.00000000
	3GP$	97822.84634868	+	3XTM	.82540000	1078.00000000
	4GP$	77853.54335907	+	4XTM	.81220000	942.00000000
	1SP$	184821.39473473	+	1XTS	.	1310.00000000
	2SP$	227115.27089688	+	2XTS	.69272451	1906.00000000
	3SP$	221015.10027978	+	3XTS	.82540000	1525.00000000
	4SP$	180426.40279319	+	4XTS	.81220000	1471.00000000
+	1XLC	80393.86953833	+	1XTC	.88280000	2500.00000000
+	2XLC	87892.61964937	+	2XTC	.86900000	3239.00000000
+	3XLC	97594.22042726	+	3XTC	.85520000	2907.00000000
+	4XLC	86533.64946388	+	4XTC	.84140000	2721.00000000
+	1XTM	14.74488639	+	1XT8	.	4050.00000000
+	2XTM	101.60776658	+	2XTB	.06696278	5181.00000000
+	1XTS	57.03378220	+	3XT8	.11120017	4984.00000000
+	1XTB	3.82833635	+	4XT8	.28726263	3983.00000000

Chapter 5
Appendix B

Optimum Model 02, Minimum Overdraft

4BST	17405.00000000		4XSJ	.01224224-	966.00000000-
4BSJ	1382.00000000		1XSC	.46495736-	8040.00000000-
4BKB	72395.00000000		2XSC	.02929207-	8040.00000000-
4BKT	646.00000000		3XSC	.01861912-	8040.00000000-
4BTT	5136.00000000		4XSC	.00950596-	8040.00000000-
4BBC	46686.00000000		1XFB	.50921075-	175100.00000000
4BBT	15915.00000000		2XFB	.01556569-	175100.00000000
4BMC	3730.00000000		3XFB	.01448276-	175100.00000000
4BMT	240.00000000		4XFB	.02000000-	175100.00000000
1RTM	731.01585626	-	1XFM	.48777093-	89550.00000000-
2RTM	1363.41655711	-	2XFM	.00112752-	39800.00000000-
3RTM	1078.00000000	-	3XFM	.	.
4RTM	942.00000000	-	4XFM	.00572000-	.
1RTS	1062.01732147	+	1XLC	.48921075	12497.55512408
2RTS	1906.00000000	+	2XLC	.	12497.55512408
3RTS	1525.00000000	+	3XLC	.	12497.55512408
4RTS	1471.00000000	+	4XLC	.	12497.55512408
1RTC	2407.91866568		1XCA	.50921075	49793.00000000-
2RTC	3239.00000000		2XCA	.01556569	85166.00000000-
3RTC	2907.00000000		3XCA	.01448276	109931.00000000-
4RTC	2721.00000000		4XCA	.02000000	147909.00000000-
1RTB	1948.23356394		4FJT	.02233886-	12169.00000000
2RTB	5181.00000000		4FST	.02118722-	17405.00000000
3RTB	4984.00000000		4FSJ	.02133423-	1382.00000000
4RTB	3983.00000000		4FKB	.01708181-	72395.00000000
3ETM	136.66204956		4FKT	.03213224-	646.00000000
4ETM	605.14484918		4FTT	.03308996-	5136.00000000
2ETS	44.79134288		4FBC	.01492594-	46686.00000000
3ETS	213.13925234		4FBT	.05083877-	15915.00000000
4ETS	619.19118817		4FMC	.02775390-	3730.00000000
2ETC	817.30386471		4FMT	.02678464-	240.00000000
3ETC	799.75159090	+	1DJT	.34035824	67715.00000000
4ETC	1710.33357670	+	1DST	.22383799	55518.00000000
2ETB	27.84066775	+	1DSJ	.14500459	.
3ETB	403.05106229	+	1DKB	.19986373	164898.00000000
4ETB	1667.04636967	+	1DKT	.47718059	3972.00000000
1GUS	33492.79664705	+	1DTT	.18282354	1591.00000000
2GUS	54981.01176138	+	1DBC	.15675806	217939.00000000
3GUS	49657.06821639	+	1DBT	.28745089	13531.00000000
4GUS	57840.83410294	+	1DMC	.09983476	17315.00000000
1RRM	24372.59310846	+	1DMT	.34001587	974.00000000
2RRM	46348.97194833	+	2DJT	.17448831	97539.00000000
3RRM	44108.68191822	+	2DST	.08618109	94846.00000000
4RRM	49993.99884166	+	2DSJ	.02637879	.
1WGJ	143301.00000000	+	2DKB	.10398333	179862.00000000
2WGJ	237901.00000000	+	2DKT	.16638561	4405.00000000
3WGJ	225099.00000000	+	2DTT	.01927531	6500.00000000
4WGJ	171872.00000000	+	2DBC	.07501306	212012.00000000

Chapter 5
Appendix C

Optimum Model 09
Optimal Primal-Dual Solution

	J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0	00000	267629.46837904	1PET	1.00000000	.
0	00000	288751.69621804	2NSP	.	.
0	00000	267629.46837904	2PET	.	.
0	00000	.	1PIN	.	.
0	00000	.	2PIN	.	.
0	00000	.	4OLC	.	.
	1GP$	93856.89374689	1GPF	1.00000000-	6785.60000000-
	2GP$	101679.30712509	2GPF	1.00000000-	6785.60000000-
	3GP$	97822.84634868	3GPF	1.00000000-	6785.60000000-
	4GP$	77853.54335907	4GPF	1.00000000-	6785.60000000-
	1SP$	184821.39473473	1SPN	.	8990.00000000-
	2SP$	227115.27089688	2SPN	.	8990.00000000-
	3SP$	221015.10027978	3SPN	.	8990.00000000-
	4SP$	180426.40279319	4SPN	.	8990.00000000-
-	1XFM	29415.33507342	1M1L	.76546288	485.70000000
	1PKB	129042.00000000	1SOL	.07456489	346.90000000
	1PST	20431.00000000	1C1L	.94302875	471.90000000
	1PJT	49090.78985040	1B1L	.06676762	625.50000000
	2PST	94846.00000000	2M1L	.14705154	557.70000000
	2PTB	5181.00000000	2SOL	.88301273	400.90000000
	2FP$	172671.23466337	2C1L	.91314960	552.90000000
	2RTC	3239.00000000	2B1L	.12035143	691.50000000
	4SBC	198342.00000000	3M1L	.85461030	509.70000000
	3RTC	2907.00000000	3SOL	.85461030	364.90000000
	3RTB	4984.00000000	3C1L	.88358615	498.90000000
	3FP$	194128.05202161	3B1L	.13185554	647.50000000
	4FP$	155542.83856615	4M1L	.82612576	485.70000000
	4RTC	2721.00000000	4SOL	.82612576	346.90000000
	4PKB	206675.00000000	4C1L	.85399594	471.90000000
	1FTC	265.81342198	4B1L	.23985866	625.50000000
	1SJT	67715.00000000	1TJT	.	27276.00000000
	1SST	55518.00000000	1TST	.00083469	35087.00000000
	1PSJ	.	1TSJ	.00099788	.
	1FP$	123456.34777089	1TKB	.00070170	35856.00000000
	1WGJ	143301.00000000	1TKT	.00221458	2731.00000000
	1RTT	1591.00000000	1TTT	.	3182.00000000
	1SBC	217939.00000000	1TBC	.00142207	17340.00000000
	1SRT	13531.00000000	1TBT	.	1238.00000000
	1SMC	17315.00000000	1TMC	.00149109	12155.00000000
	1RMT	3233.00000000	1TMT	.	1063.00000000
	2PJT	88887.21014960	2TJT	.00066510	27276.00000000
	2SST	94846.00000000	2TST	.00137283	35087.00000000
	2PSJ	.	2TSJ	.00154470	.
	2SKB	179862.00000000	2TKB	.	35856.00000000
	2RKT	4405.00000000	2TKT	.00233778	2731.00000000
	2STT	6500.00000000	2TTT	.00247426	3182.00000000
	4ETM	481.70089896	2TBC	.00006093	17340.00000000

2PBT	30311.34405236	2TBT	.	1238.00000000
2SMC	16595.00000000	2TMC	.00025677	12155.00000000
2WGJ	237901.00000000	2TMT	.	1063.00000000
3PJT	83964.00000000	3TJT	.00150867	27276.00000000
3SST	85063.00000000	3TST	.00187777	35087.00000000
4PSJ	57531.00000000	3TSJ	.00202897	.
3SKB	171315.00000000	3TKB	.00181024	35856.00000000
3PKT	4164.00000000	3TKT	.00224493	2731.00000000
3SKT	4164.00000000	3TTT	.	3182.00000000
2SBC	212012.00000000	3TBC	.00199299	17340.00000000
2BBT	39681.22109890	3TBT	.	1238.00000000
3SMC	20130.00000000	3TMC	.	12155.00000000
3PTT	25155.00000000	3TMT	.00141631	1063.00000000
4BJT	12169.00000000	4TJT	.01536477	27276.00000000
4BST	17405.00000000	4TST	.01447301	35087.00000000
4BSJ	1382.00000000	4TSJ	.01451834	.
4BKB	72395.00000000	4TKB	.01381633	35856.00000000
4BKT	646.00000000	4TKT	.01845760	2731.00000000
4BTT	5136.00000000	4TTT	.01300735	3182.00000000
4BBC	46686.00000000	4TBC	.01298650	17340.00000000
4BBT	15915.00000000	4TBT	.02327967	1238.00000000
4BMC	3730.00000000	4TMC	.01503469	12155.00000000
4BMT	240.00000000	4TMT	.01818435	1063.00000000
3PBC	219495.00000000	1SUG	.05747994	687.00000000-
3PBT	23274.77890110	2SUG	.04293959	687.00000000-
4ETC	1462.98418140	3SUG	.02859917	687.00000000-
4SJT	58402.00000000	4SUG	.01419878	687.00000000-
1PBT	53812.87704654	1ORM	.04293959	1064.00000000
3SBC	219495.00000000	2ORM	.02859917	1064.00000000
3WGJ	225099.00000000	3ORM	.01419878	1064.00000000
4PST	56624.00000000	4ORM	.	1064.00000000
1SKT	3972.00000000	1XWJ	.04171758	.
2SMT	2461.00000000	2XWJ	.04114397	.
3SMT	772.00000000	3XWJ	.04056795	.
4SKT	2086.00000000	4XWJ	.04000000	.
1SKB	164898.00000000	1XWC	.03128819	.
2PKB	232324.46946315	2XWC	.03085798	.
3PKB	118852.53053685	3XWC	.03042596	.
4SKB	134280.00000000	4XWC	.03000000	.
1SJ$	65072.90221039	1XSJ	.04347462-	966.00000000-
2SJ$	106481.59895045	2XSJ	.03029404-	966.00000000-
3SJ$	99897.92336018	3XSJ	.01777829-	966.00000000-
4SJ$	72870.15832090	4XSJ	.00709939-	966.00000000-
1SC$	124588.21773869	1XSC	.03554376-	8040.00000000-
2SC$	127618.38041844	2XSC	.02288636-	8040.00000000-
3SC$	127784.38919864	3XSC	.01326938-	8040.00000000-
4SC$	112626.95421922	4XSC	.00454361-	8040.00000000-
1RTS	1283.32761710	1XFB	.01454035-	175100.00000000

	4GUS	54935.45983964		2XFB	.01434042-	175100.00000000	
	3WGC	410940.00000000		3XFB	.01440039-	175100.00000000	
	4WGC	362707.00000000		4XFB	.01419878-	175100.00000000	
	1FB$	203294.64482531	-	1XFM	.	139550.00000000	-
	2FB$	216948.20773871	-	2XFM	.	89550.00000000	-
	3FB$	204047.37706665	-	3XFM	.	50000.00000000	-
	4FB$	196708.11869372	-	4XFM	.	50000.00000000	-
	1LB$	.	+	1XLC	.01454035	.	
	2LB$	.	+	2XLC	.01434042	.	
	3LB$	.	+	3XLC	.01440039	.	
	4LB$	.	+	4XLC	.01419878	.	
	1WGC	400152.00000000		1XCA	.01454035	49793.00000000	-
	2SJT	97539.00000000		2XCA	.01434042	85166.00000000	-
	3SJT	83964.00000000		3XCA	.01440039	109931.00000000	-
	2RTM	1419.00000000		4XCA	.01419878	147909.00000000	-
	4PJT	70571.00000000		4FJT	.01536477-	12169.00000000	
	4RRM	45212.48190325		4FST	.01447301-	17405.00000000	
	3PSJ	.		4FSJ	.01451834-	1382.00000000	
	4RTS	1471.00000000		4FKB	.01381633-	72395.00000000	
	4PKT	2732.00000000		4FKT	.01845760-	646.00000000	
	4PMT	332.00000000		4FTT	.01300735-	5136.00000000	
	2RRM	46286.09574288		4FBC	.01298650-	46686.00000000	
	4RTB	3983.00000000		4FBT	.02327967-	15915.00000000	
	4PMC	33786.62992290		4FMC	.01503469-	3730.00000000	
	3RMC	28.37007710		4FMT	.01818435-	240.00000000	
	1RTB	4046.17166365	+	1DJT	.18667828	67715.00000000	
	1RTC	2500.00000000	+	1DST	.09696658	55518.00000000	
	1SSJ	.	+	1DSJ	.03692601	.	
	1RTM	930.00000000	+	1DKB	.10930863	164898.00000000	
	1PKT	1241.00000000	+	1DKT	.18823556	3972.00000000	
	1STT	1591.00000000	+	1DTT	.04780122	1591.00000000	
	1PBC	200599.00000000	+	1DBC	.07763738	217939.00000000	
	1RRM	30394.35061202	+	1DBT	.13480094	13531.00000000	
	1PMC	5160.00000000	+	1DMC	.02832851	17315.00000000	
	1SMT	974.00000000	+	1DMT	.15120716	974.00000000	
	2RTS	1906.00000000	+	2DJT	.18117183	97539.00000000	
	2GUS	56854.93732424	+	2DST	.09314430	94846.00000000	
	2SSJ	.	+	2DSJ	.03381841	.	
-	2XFM	32401.96468818	+	2DKB	.10636486	179862.00000000	
	2SKT	4405.00000000	+	2DKT	.18074356	4405.00000000	
	2PTT	4909.00000000	+	2DTT	.04358449	6500.00000000	
	2FTC	907.12302301	+	2DBC	.07605748	212012.00000000	
	2SBT	32150.00000000	+	2DBT	.12882644	32150.00000000	
	2PMC	16595.00000000	+	2DMC	.02706648	16595.00000000	
	1PMT	3144.00000000	+	2DMT	.14581985	2461.00000000	
	3RTS	1525.00000000	+	3DJT	.17663955	83964.00000000	
	3PST	85063.00000000	+	3DST	.09012640	85063.00000000	
	3SSJ	.	+	3DSJ	.03149603	.	

	3RTM	1078.00000000	+	3DKB	.10370020	171315.00000000
	3RRM	42931.31755851	+	3DKT	.17389570	4164.00000000
	3STT	19177.00000000	+	3DTT	.04208604	19177.00000000
	3ETM	5.52341316	+	3DBC	.07394268	219495.00000000
	3SBT	31959.00000000	+	3DBT	.12318463	31959.00000000
	1BBT	41519.87704654	+	3DMC	.02533659	20130.00000000
	2BMT	772.00000000	+	3DMT	.14073562	772.00000000
	4RTM	942.00000000	+	4DJT	.17379011	58402.00000000
	4SST	39219.00000000	+	4DST	.08833431	39219.00000000
	3BTT	5978.00000000	+	4DSJ	.03030194	56149.00000000
	3GUS	45904.34662807	+	4DKB	.10312671	134280.00000000
	4WGJ	171372.00000000	+	4DKT	.16837957	2086.00000000
	4STT	842.00000000	+	4DTT	.03877285	842.00000000
-	4XEM	35848.05542775	+	4DBC	.07399555	198342.00000000
	4SBT	15082.00000000	+	4DBT	.11844731	15082.00000000
	4SMC	30085.00000000	+	4DMC	.02356802	30085.00000000
	4SMT	92.00000000	+	4DMT	.13789035	92.00000000
	1BJT	8651.78985040	+	1XTM	.69089799	930.00000000
	2ETS	118.53311618	+	2XTM	.09166192	1419.00000000
	3ETS	40.88126299	+	3XTM	.81815543	1078.00000000
	4ETS	496.39710859	+	4XTM	.80866126	942.00000000
+	1XTS	26.67238290	+	1XTS	.	1310.00000000
	2BKB	52462.46946315	+	2XTS	.82762311	1906.00000000
	3ETC	599.38707159	+	3XTS	.81815543	1525.00000000
	1GUS	38276.46693582	+	4XTS	.80866126	1471.00000000
	3BBT	30997.00000000	+	1XTC	.86791632	2500.00000000
	2BBC	212012.00000000	+	2XTC	.85790747	3239.00000000
-	3XEM	45519.75300430	+	3XTC	.84800538	2907.00000000
	4BBC	245028.00000000	+	4XTC	.83809331	2721.00000000
+	1XTB	3.82833635	+	1XTB	.	4050.00000000
	2WGC	408469.00000000	+	2XTB	.07155186	5181.00000000
	3BMC	20158.37007710	+	3XTB	.10085668	4984.00000000
	4SSJ	56149.00000000	+	4XTB	.22665379	3983.00000000

	4GUS	57840.83410294		2XFB	.01434042-	175100.00000000
	3WGC	410940.00000000		3XFB	.01440039-	175100.00000000
	4WGC	362707.00000000		4XFB	.01419878-	175100.00000000
	1FB$	195381.77216666	-	1XFM	.48860620-	102221.79226793-
	2FB$	201625.51275968	-	2XFM	.	52221.79226793-
	3FB$	193146.92133069	-	3XFM	.	12671.79226793-
	4FB$	192122.45470223	-	4XFM	.	12671.79226793-
	1LB$	.	+	1XLC	.50995855	.
	2LB$	.	+	2XLC	.01434042	.
	3LB$	.	+	3XLC	.01440039	.
	4LB$	.	+	4XLC	.01419878	.
	1WGC	400152.00000000		1XCA	.50995855	49793.00000000-
	2SJT	97539.00000000		2XCA	.01434042	85166.00000000-
	3SJT	83964.00000000		3XCA	.01440039	109931.00000000-
	2RTM	1419.00000000		4XCA	.01419878	147909.00000000-
	4PJT	70571.00000000		4FJT	.02159083-	12169.00000000
	4RRM	49993.99884166		4FST	.02046646-	17405.00000000
	3PSJ	.		4FSJ	.02061194-	1382.00000000
	4RTS	1471.00000000		4FKB	.01638624-	72395.00000000
	4PKT	2732.00000000		4FKT	.03128064-	646.00000000
	4PMT	332.00000000		4FTT	.03263485-	5136.00000000
	2RRM	47456.71331330		4FBC	.01434987-	46686.00000000
	4RTB	3983.00000000		4FBT	.04978663-	15915.00000000
	4PMC	33815.00000000		4FMC	.02716359-	3730.00000000
	4PBT	30997.00000000		4FMT	.02598733-	240.00000000
	1RTB	1948.23356394	+	1DJT	.33932429	67715.00000000
	1RTC	2407.91866568	+	1DST	.22333912	55518.00000000
	1SSJ	.	+	1DSJ	.14483356	.
	1RTM	731.01585626	+	1DKB	.19933742	164898.00000000
	1PKT	1241.00000000	+	1DKT	.47545769	3972.00000000
	1STT	1591.00000000	+	1DTT	.18267868	1591.00000000
	1PBC	200599.00000000	+	1DBC	.15647956	217939.00000000
	1RRM	24372.59310846	+	1DBT	.28682750	13531.00000000
	1PMC	5160.00000000	+	1DMC	.09996647	17315.00000000
	1SMT	974.00000000	+	1DMT	.33914966	974.00000000
	2RTS	1906.00000000	+	2DJT	.17305887	97539.00000000
	2GUS	56328.55491205	+	2DST	.08531254	94846.00000000
	2SSJ	.	+	2DSJ	.02585382	.
-	2XFM	10396.45193513	+	2DKB	.10312248	179862.00000000
	2SKT	4405.00000000	+	2DKT	.16390247	4405.00000000
	2PTT	4909.00000000	+	2DTT	.01882357	6500.00000000
	2ETC	958.30103106	+	2DBC	.07440847	212012.00000000
	2SBT	32150.00000000	+	2DBT	.09603380	32150.00000000
	2PMC	16595.00000000	+	2DMC	.01112325	16595.00000000
	1PTT	.	+	2DMT	.13511484	2461.00000000
	3RTS	1525.00000000	+	3DJT	.16883085	83964.00000000
	3PST	85063.00000000	+	3DST	.08260944	85063.00000000
	3SSJ	.	+	3DSJ	.02385347	

	4GUS	57840.83410294		2XFB	.01896522-	175100.00000000
	3WGC	410940.00000000		3XFB	.34612496-	175100.00000000
	4WGC	362707.00000000		4XFB	.01419878-	175100.00000000
	1FB$	195381.77216666	-	1XFM	.05789261-	102221.79226793-
	2FB$	197526.41477165	-	2XFM	.	37726.24234476-
	3FB$	190494.97291849	-	3XFM	.32708043-	9072.15715246
	4FB$	192146.38748919	-	4XFM	.	9590.21336772-
	1LB$	.	+	1XLC	.07792936	.
	2LB$	.	+	2XLC	.01896522	.
	3LB$	.	+	3XLC	.34612496	.
	4LB$	.	+	4XLC	.01419878	.
	1WGC	400152.00000000		1XCA	.07792936	49793.00000000-
	2SJT	97539.00000000		2XCA	.01896522	85166.00000000-
	3SJT	83964.00000000		3XCA	.34612496	109931.00000000-
	2RTM	1294.87162799		4XCA	.01419878	147909.00000000-
	4PJT	70571.00000000		4FJT	.02159083-	12169.00000000
	4RRM	49993.99884166		4FST	.02046646-	17405.00000000
	3PSJ	.		4FSJ	.02061194-	1382.00000000
	4RTS	1471.00000000		4FKB	.01638624-	72395.00000000
	4PKT	2732.00000000		4FKT	.03128064-	646.00000000
	4PMT	332.00000000		4FTT	.03263485-	5136.00000000
	2RRM	44982.91670934		4FBC	.01434987-	46686.00000000
	4RTB	3983.00000000		4FBT	.04978663-	15915.00000000
	4PMC	33815.00000000		4FMC	.02716359-	3730.00000000
	4PBT	30997.00000000		4FMT	.02598733-	240.00000000
	1RTB	1948.23356394	+	1DJT	.27358142	67715.00000000
	1RTC	2407.91866568	+	1DST	.15315479	55518.00000000
	1SSJ	.	+	1DSJ	.07376581	.
	1RTM	731.01585626	+	1DKB	.16253987	164898.00000000
	1PKT	1241.00000000	+	1DKT	.33622155	3972.00000000
	1STT	1591.00000000	+	1DTT	.08768916	1591.00000000
	1PBC	200599.00000000	+	1DBC	.11684577	217939.00000000
	1RPM	24372.59310846	+	1DBT	.20337067	13531.00000000
	1PMC	5160.00000000	+	1DMC	.04532251	17315.00000000
	1SMT	974.00000000	+	1DMT	.23186778	974.00000000
	2RTS	1882.31560432	+	2DJT	.24254372	97539.00000000
	2GUS	53319.23543078	+	2DST	.12672396	94846.00000000
	2SSJ	.	+	2DSJ	.05080681	.
+	2XTM	124.12837201	+	2DKB	.14372904	179862.00000000
	2SKT	4405.00000000	+	2DKT	.28399061	4405.00000000
	2PTT	4909.00000000	+	2DTT	.06040966	6500.00000000
	2ETC	643.42761176	+	2DBC	.09916410	212012.00000000
	2SBT	32150.00000000	+	2DBT	.17280671	32150.00000000
	2PMC	16595.00000000	+	2DMC	.03095448	16595.00000000
	1BBC	.	+	2DMT	.19773691	2461.00000000
	3RTS	1525.00000000	+	3DJT	.26171244	83964.00000000
	3PST	85063.00000000	+	3DST	.15809663	85063.00000000
	3SSJ	.	+	3DSJ	.08694929	.

Chapter 5
Appendix D

Revised Right Hand Side Values
Equations 2-1, 11-2A and 13-1

Equation (2-1): (hours)

Row ID:	Period (1)	(2)	(3)	(4)
M1L	485.7	557.7	509.7	485.7
S0L	306.9	354.9	322.9	306.9
C1L	411.9	483.9	435.9	411.9
B1L	665.5	737.5	689.5	665.5

Equation (11-2A): (pounds)

Row ID:	Quantity	Row ID	Quantity
4FJT	26995	4FTT	203
4FST	9874	4FBC	80645
4FSJ	51181	4FBT	8677
4FKB	70632	4FMC	4943
4FKT	1473	4FMT	573

Equation (13-1): (hours)

Row ID:	Period (1)	(2)	(3)	(4)
XTM	960	1104	1008	960
XTS	1600	1840	1680	1600
XTC	2560	2944	2688	2560
XTB	3840	4416	4032	3840

Chapter 5
Appendix E

Planning Models 07 through 011
Revised Right Hand Side Values
Equations 1, 1-1, 2-1, 8-3, 12-1, 13-1

Equations (1) and (1-1): (pounds)
Finished goods inventory, 1-1-66:

Row ID:	Quantity	Row ID:	Quantity
TJT	7606	TTT	8237
TST	34214	TBC	32945
TSJ	0	TBT	9859
TKB	34182	TMC	5805
TKT	3355	TMT	978

(In equation (1) quantities shown apply to T_{jT} , T = 1,M.
In equation (1-1) they apply to T = 1,
and T_{jT} = 0 for T = 2,M.)

Equation (2-1): (hours)

Row ID:	Quantity
1-10M1L	509.7
1-10S0L	322.9
1-10C1L	435.9
1-10B1L	689.5

Equation (13-1): (hours)

Row ID:	Quantity
1-10XTM	1008
1-10XTS	1680
1-10XTC	2688
1-10XTB	4032

Equation (8-3): ($)

Row ID:	Quantity	Row ID:	Quantity
1XCA -	6504	6XCA -	209255
2XCA -	19279	7XCA -	270249
3XCA -	43842	8XCA -	326891
4XCA -	104970	9XCA -	383845
5XCA -	157586	10XCA -	439636

Equation (12-1): (pounds)
Actual sales for January - June 1966

	Period					
Row ID:	(1)	(2)	(3)	(4)	(5)	(6)
DJT	47140	59494	55105	62891	83673	75298
DST	53558	43755	50845	44630	74223	78949
DSJ	0	0	0	0	0	0
DKB	128834	93986	170803	126483	136093	182001
DKT	654	3062	5015	5554	14025	11136
DTT	9048	3401	4524	3557	2371	2246
DBC	121320	145860	180300	171917	217984	253159
DBT	25038	11689	22266	29449	22888	26093
DMC	14350	18570	11555	31939	18285	16965
DMT	406	517	3527	10839	6197	2941

Chapter 6

COMPB: THE SECOND TEST

COMPB is the second firm studied in depth as a part of this research. A thorough analysis of COMPB isolated the fundamental relationships and characteristics required to formulate a fully integrated multiperiod model of the firm. However, the absence of data in a usable form required that a number of gross averaging procedures be used to provide the necessary coefficients and right-hand side values. Largely because these values could not be precisely quantified the model did not adequately simulate the firm and it was found to be nonrepresentative. For this reason COMPB planning models were not developed and no attempt was made to trace marginal revenue product curves. The development of the COMPB fully integrated model has been completely documented and upon request the author will be pleased to provide any interested individual with additional informaton concerning it. However, the COMPB plan of study closely parallels that of COMPA and it will not be described in detail in this paper.

6.1 Company Background

COMPB is of particular interest because of its similarity to the "warehousing model" of linear programming used by Charnes, Cooper, and Miller in their development of fully integrated models.[1] In this regard COMPB purchases all merchandise and stores it until sold. No manufacturing or conversion is involved. Unlike the "warehousing model" COMPB is not limited by physical storage capacity and the only effective constraints on its operations are demand and the availability of funds.

[1] *Op. cit.*

COMPB has grown rapidly during the few years it has been in operation and continuously expanding sales, inventory and receivables have required regular infusions of funds. Because profits were modest, most funds had to be obtained from external sources. However, because funds were absorbed by inventory and receivables, COMPB has been able to maintain a reasonable level of risk in its capital structure. For this reason it has not been subjected to severe funds rationing and available working capital has generally been adequate to support the desired level of operations.

COMPB purchases merchandise for resale from both foreign and domestic sources. All merchandise is sold directly to customers at retail prices and a few items are also sold through dealers who are allowed discounts. Some seasonality is evident and demand peaks occur in both Spring and Fall.

There are very few companies that compete directly with COMPB and it has no single competitor across its entire line. Expansion of demand is dependent on the selective addition of desirable items to the catalog and the identification and solicitation of potential customers. Although increased sales must be generated by these and other demand stimulating activities, there appear to be ample opportunities for continued rapid growth. Absolute funds rationing is not apparent in COMPB. However, the possible use of a fully integrated model to allocate cash and provide information needed for better cash management in a rapidly growing firm is of sufficient interest to justify this analysis.

6.2 General Properties of the COMPB "Warehouse" Model

The COMPB fully integrated model was formulated to determine the optimal distribution of receipts, sales, and inventory balances for a group of aggregate products over a multi-period planning horizon. Therefore, primary decision variables were specified to reflect the receipt, sale, and inventory balance of each product in every period. Because of the heterogeneous product mix, variables were quantified in monetary terms.

Formulation of a fully integrated multi-period "warehouse" model requires recognition in the tie equations of all fundamental economic relationships that link receipts, sales, and inventory balances.

For this reason these constraints are largely responsible for the representativeness of the model and precise determination of all matrix coefficients is absolutely essential.

The accumulation, cash flow and other financial constraints in the COMPB model are similar in form and content to those developed for COMPA. Although the basic unit of quantification may differ among model types, the financial segments of all fully integrated models are remarkably similar. This finding indicates that the general cash flow and supporting financial constraints can be added to many existing sub-optimizing models to expand them to the fully integrated form. Imaginative adaptations will surely be required but the applicability of the concepts is clear.

The objective function in the warehouse model can be formulated to minimize or maximize any objective. However, in order to provide meaningful comparisons it is desirable to design the functional so that solution values can be related to commonly used measures of cost or profitability. In the COMPB model the functional was formulated to maximize contribution, a measure comparable to gross margin in the conventional operating statement.

COMPB readily satisfies the pre-conditions necessary to permit the use of linear programming analysis, i.e., established costs and prices, standard products, and constant returns to scale. These criteria do not seem to frustrate development of a warehouse model to the extent that they affect the more common "production" model. The typical merchandising firm, with a large and heterogeneous product line, buys and sells at set prices and only cash or small trade discounts have to be considered. In addition, linearity is seldom in question. For example, COMPB's purchases appear to have no impact on its suppliers and its customers' insensitivity to price increases indicates that the demand schedule it faces is not price elastic. Therefore, in COMPB, both total cost of purchases and total sales revenue could be adequately defined as linear functions of quantities bought and sold.

6.3 Multiple Products, Model Size, and Problems of Aggregation

As previously mentioned, COMPB's heterogeneous product mix made it necessary to quantify all decision variables in monetary terms. However, without aggregation there are about 3,000 items in COMPB's product line and it was obviously impossible to include receipt, sales and inventory balance decision variables in the model to represent each of them. For this reason it was decided to aggregate products into groups with similar markups on selling price. These groups could then be used as proxies for all individual items within the appropriate range. However, grouping of items by markup alone would have been too crude to permit the model to recognize all relevant cash flow dissimilarities and these basic groups were further broken down by domestic or foreign origin and by available credit terms.

In order to specifically define the aggregate product groups it was necessary to determine the markup on each individual item offered for sale. These data were not available in summary form and were developed as a part of this research. Rather than analyze every item in the line a sampling procedure was used and the markup analysis was based on a subpopulation consisting of all fast moving and/or high value items. Stock record cards were screened to identify items in this category and a total of 862 were found.[2] Relevant data were summarized on punch cards and a computer program was written to calculate the markup factors and summarize other information about each item in the sample. The markup deck was sorted a number of ways and checks were made to determine modal points or other indications of natural groupings. However, all practical systems of aggregation had some drawbacks and it was decided to formulate two models based on somewhat different group definitions. Both of these models are structurally identical and each has 20 receipt, inventory, and primary sales variables plus three secondary sales variables in each period.

[2] It was also found that these items accounted for 90% of total sales revenue during 1966.

Values for most coefficients used in the COMPB model are single valued estimates that summarize a given characteristic of the individual items in an aggregate group. Because individual items vary in unit price, cost, and quantities bought and sold, summary measures are weighted by these factors. Summary measures are not particularly desirable coefficients because they tend to obscure dissimilarities among alternatives. However, aggregation was an unavoidable prerequisite to formulation of the COMPB model and in this case groups can only be represented by an average of their individual values.

Weighted average coefficient values used in the COMPB simulation model had to be developed as part of this study. Although COMPB maintains considerable detailed information concerning individual catalog numbers, at the time this research was in progress there was little helpful summary data available other than that found in standard accounting records and reports. This data inadequacy may be attributable to the development of item groupings specifically for use in the fully integrated model. However, the availability of detailed stock records for each catalog number did make it possible to attempt to develop the information required. In addition, as a result of the earlier markup analysis both transactions and cost/price data were available on punch cards. This permitted computer calculation of averages and a series of programs was written to perform this task.

The weighted average coefficients derived from the sample are used as estimates of the values that would be found in a population consisting of all catalog numbers. This is not an unreasonable assumption because although only 30% of the catalog numbers are included in the sample, collectively they represent 90% of 1966 sales. However, the coefficients reflect the particular combination of mix and volume that occurred during 1966. If these values are used in a planning model the assumption must be made that they will also be valid estimates of future relationships.

With minor exceptions only aggregate actual receipt, sales and inventory information was available in COMPB's records. That is, the usual summary postings from sales and purchase journals to

general ledger accounts. These accounts had to be disaggregated on the basis of the 20 groups before the model could be tested by simulation. There was no way to accurately disaggregate this information so a procedure was devised to extrapolate the breakdown of receipts and sales in the sample to the total quantities. This required the assumption that actual receipts and sales in each period were distributed among the groups in the same proportions as those in the sample. Although this assumption is tenuous, the large proportion of both actual sales and receipts that were accounted for in the sample appeared to make it tolerable. These determined values were the parameters used to force the model to simulate COMPB's experience during the period 1 July through 31 December 1966.

The aggregation and averaging that has been required in the formulation of the COMPB model indicates a fundamental difference in the type of data needed by the "warehousing model" and that normally of importance to a merchandising firm. In order for a firm to provide definite values for coefficients required in the fully integrated model it would have to follow a very rigid pricing policy. Although all items would not be required to have identical markups, only a few price lines could be used and every item in a line would have to be marked-up by the same factor. All purchase and sales information would have to be accumulated by price line and where necessary by subclassification to reflect different trade credit terms. Planning, budgeting, and stock control would also have to be by price line. Because it does not appear practical to tie all aspects of planning and control to price lines it seems unlikely that a firm would be willing to accept such a system as a prerequisite to the development of a representative linear programming model. If this is true, the only alternative is to use item groups especially aggregated for the model and accept the weaknesses inherent in the summary measures that must be used to represent important group characteristics.

6.4 Results of the Simulation Tests

Development of constraints and determination of coefficients established that information available in COMPB was adequate to permit formulation of a fully integrated model of the firm. The

constraints represent fundamental relationships and their formulation is not particularly dependent on detailed information. However, estimation of coefficient values required numerous compromises. To determine if the estimates were adequate to permit the COMPB models to simulate the six-month test period, solutions were obtained and values compared with actual experience. The test hypothesis was: "If the model is forced purchase and sell the quantity and mix of merchandise actually purchased and sold during the test period, predicted levels of profit, inventory, sales revenue and borrowing should approximate actual."

The COMPB simulation models were solved using the LP-90 computer code. Including 6 periods and 20 item groups in the analysis made the problem quite large and each initial tableau, including slack vectors and the functional was 475 X 621 with 2,282 non-zero elements or a density of .774%. Test results were evaluated in the same manner as those from the COMPA simulation in Chapter 4. Table 6.1 compares a few selected values from the solutions to the two simulation models with appropriate actual values from COMPB's records. These comparisons are generally self-explanatory but in certain cases apparent representativeness may be illusory. All estimates made by the models with the exception of inventory balances and gross margin are directly or indirectly related to total receipts or total sales. These parameters were stipulated in the simulation models. In addition, lags between receipts or sales and resulting cash flows make several estimates dependent on information from periods that precede the planning cycle. Values in Table 6.1 that are heavily or completely dependent on information supplied through the right-hand side are marked with an asterisk.

Most comparisons shown in Table 6.1 indicate that estimates developed by the simulation models closely approximate actual experience. This result establishes the validity of coefficients that functionally relate particular variables to total sales or total receipts. For example, model estimates of end of period accounts receivable balances are very close to actual. However, the most significant test of the COMPB model is its ability to simulate inventory investment and gross margin. Comparisons in Table 6.1 show that both estimates are considerably in excess of actual. This indicates that inaccurate

Table 6.1
COMPB

Comparison of Financial Estimates Made by Simulation Models 01 and 02 with Actual Experience during July-December 1966

(000 omitted)	Actual	Model 01	% diff.	Model 02	% diff.
Gross margin	$ 585.8	$ 649.4	10.6	$ 649.0	10.6
Less: interest	25.7	19.2		19.3	
From functional	$ 560.1	$ 630.2	12.4	$ 629.7	12.3
Inventory, EOM:					
July	$ 399.6	$ 404.5	1.2	$ 404.8	1.2
August	427.6	445.0	4.0	446.0	4.2
September	420.8	443.9	5.5	446.2	6.0
October	440.4	470.2	7.0	473.5	7.5
November	472.6	509.8	8.6	512.3	8.2
December	508.9	555.6	9.2	560.9	10.2
Payments to suppliers:					
July	$ 98.3	$ 98.3*	-	$ 98.3*	-
August	108.3	108.3*	-	108.3*	-
September	101.7	101.7*	-	101.7*	-
October	123.8	123.6	-	123.6	-
November	80.7	94.7	17.2	94.7	17.2
December	134.4	108.6	19.2	108.6	19.2
Total	$ 647.1	$ 635.2	1.7	$ 635.2	1.7
Accounts receivable balance, EOM:					
July	$457,887	$459,061*	-	$458,328*	-
August	427,546	427,536*	-	427,115*	
September	506,040	502,098	1.0	512,272	1.5
October	496,139	492,001	1.0	496,957	-
November	475,608	471,444	1.0	472,121	.5
December	453,500	450,779	.5	446,599	1.5
Accounts payable balance, EOM:					
July	$247,348	$247,351*	-	$247,408*	-
August	251,109	250,321	-	250,355	-
September	256,708	255,445	.5	255,447	.5
October	249,440	242,426	2.5	242,391	2.5
November	293,090	271,565	7.5	271,530	7.5
December	283,451	284,420	.4	284,466	.4

Table 6.1 (continued)

Factor loan, EOM:					
July	$329,972	$335,969	1.9	$336,362	1.9
August	264,530	272,490	3.0	274,524	3.5
September	41,208	38,554	6.5	42,207	2.3
October	0	0		0	
November	0	0		0	
December	0	0		0	
Line of credit, EOM:					
October	$100,000	$ 87,535	12.5	$ 87,693	12.3
November	100,000	87,535	12.5	87,693	12.3
December	100,000	87,535	12.5	87,693	12.3
Cash balance, EOM:					
July	($ 26,205)	($ 22,500)*		($ 22,500)*	
August	(28,090)	(22,500)*		(22,500)*	
September	(22,500)	(22,500)*		(22,500)*	
October	31,900	0		0	
November	69,500	57,994		60,345	
December	32,000	42,913		42,052	

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* A parameter value, not model determined

coefficients are forcing the tie equations to accumulate too much inventory. However, because of the averaging process used to develop the coefficients it is impossible to determine the magnitude or distribution of these errors or isolate variations attributable to misallocation of receipts and sales to item groups and periods.

The overvaluation of inventory is of no particular consequence in the simulation tests. However, if the models were used for planning purposes it would flow through as a reduction in total receipts required to support the specified levels of sales and inventory. As a result all estimates functionally related to total receipts would also be understated. Therefore, it seems unlikely that the COMPB model could provide useful estimates of the financing required to support specified patterns of sales. For this reason no useful purpose would have been served by conducting the planning test phase of the study and testing of the COMPB model was terminated.

The fact that a model of COMPB could be developed in the warehouse context adds validity to the earlier finding that it may be possible to formulate a fully integrated, multiperiod model from data available in the firm. However, the COMPB model failed the test of representativeness. This can be attributed primarily to the substantial aggregation that was required and to the lack of data in usable form. It seems unlikely that a representative model can be developed when available data have to be aggregated, disaggregated, and manipulated to the extent necessary in COMPB.

In future applications to warehousing firms the ultimate usability of the fully integrated model will be dependent to a large extent on the feasibility of preplanning and structuring of data to conform to model requirements. If there are a large number of items in inventory and considerable aggregation is necessary, it appears certain that a representative fully integrated model cannot be developed unless data needs are provided for in advance.

Chapter 7

SUMMARY

The purpose of this research has been to test and evaluate the practical usefulness of linear programming as a management tool for short term financial planning. During the last few years a number of management scientists and financial theorists have proposed ways in which linear programming might be used for short term financial planning in the context of the total enterprise. These proposals have clearly shown that short term returns to the firm can be optimized if financial and operational planning are integrated. Linear programming analysis appears well suited to this task because of its ability to develop optimal plans that recognize all specified limitations and interdependencies.

Two actual firms have been used as subjects for the tests in this study. These firms were asked to cooperate because they appeared to satisfy the linear programming economic criteria and other prerequisites that had been established. The first, COMPA, was of particular interest because it operates under severe funds rationing and the similarity of the second, COMPB, to the "warehousing model" justified its inclusion in the analysis.

The objectives of this study were threefold. The first was to determine if the information that is routinely accumulated in a firm can be used to formulate a fully integrated linear programming model that has the same general properties as the models developed in theoretical applications. Linear programming cannot be used for financial or integrated planning unless a representative fully integrated model is available. However, theorists have devoted little attention to the practical difficulties attendant to the development of models in real firms.

The second objective was to establish that the models developed from data available in COMPA and COMPB were representative of the firms. If the models were shown to be representative they could be used for planning purposes. The third objective was to validate or qualify the usefulness of the representative fully integrated linear programming models for short-term financial planning.

Before linear programming analysis is feasible it must be established that the economic characteristics of a firm satisfy certain criteria. That is, a firm must have discreet costs for each input, fixed prices for every product and approximately linear returns to scale, at least in the normal operating range. Most linear programming analyses proceed on the assumption that all important functions can be approximated by linear expressions, but before linear approximations can be used with confidence it is necessary to establish the magnitude of errors that can be expected. In COMPA a series of simple regression analyses were used to show that linear relationships approximated those actually observed. However, in COMPB it was necessary to assume that both supply and demand schedules were horizontal. Although this assumption appeared to be supported by discernible facts, if the actual shape of a function is completely unknown it is unlikely that it can be approximated by a linear expression with any degree of assurance.

The economic criteria exclude some types of businesses from linear programming analysis and thus limit the applicability of the technique. Nonetheless there are many firms that can be represented by fully integrated models. Although in this study practical limitations excluded large and/or complex concerns, programming theory can be extended without regard for size. Development of a fully integrated model to represent a large complicated firm is probably dependent only on the skill and ingenuity of the individuals responsible. However, computational restrictions do establish practical limits. In developing models of both COMPA and COMPB the LP-90 capacity of 1,025 rows was a constant consideration. Newer codes and decomposition routines have expanded available capacity but for the foreseeable future computational limitations will continue to be operational.

Fully integrated models of both COMPA and COMPB were developed in this study. The successful formulation of these models establishes that if the economic characteristics of a firm satisfy the linear programming criteria it may be possible to formulate a fully integrated, multi-period model from data available in the firm. This finding supports the conclusion that linear programming is a feasible technique for short-term financial planning. It should not be concluded that fully integrated models can be easily formulated or that data are necessarily available in usable form. Rather in both firms studied a considerable amount of information had to be developed as a part of this research. Usually the data requirements of the models appeared reasonable and inadequacies were attributable to incomplete record keeping by the firm. In other instances information was needed that would not normally be found in any firm.

After the models were formulated they were tested for representativeness. Simulation tests showed that both the COMPA fully integrated comprehensive and aggregate models adequately approximated actual experience and could be used for planning purposes. The COMPB model was not adequately representative and it was excluded from the planning test phase. The primary shortcomings of the COMPB model were attributable to the amount of product aggregation that had been required. If large numbers of variables are to be grouped, as in COMPB, a representative fully integrated model can probably be successfully developed only if pricing policy and data accumulation are structured to supply the necessary aggregate information. Because of the difficulties and uncertainties associated with aggregation of products, the usability of linear programming for financial planning can be significantly extended by additional research to develop systematic ways to reduce model size without proportionate losses in representativeness.

The final phase of the tests used the COMPA model to solve a series of hypothetical "what if" problems. Most of the plans produced by these models were realistic and the important differences between them were easily identified. The capability of linear programming analysis to evaluate

complex and interrelated proposals and provide meaningful information was particularly apparent in the ten period models. Evidence provided by the planning tests clearly shows that a fully integrated linear programming model can provide financial information that management should find useful for short-term planning and decision making.

Linear programming has been found to be useful for short-term financial planning and if a representative fully integrated model is developed it can be used to expand both the range and depth of financial analysis that it is practical for management to expect. This does not mean that management can delegate decision making to the model. However, linear programming can be used to evaluate proposals and develop information that could not otherwise be obtained. The usefulness of the technique appears to lie in this area rather than in developing optimal operating and/or financial plans that can be directly implemented. Because planning models can realistically simulate actual operating situations, they give management the capability to consistently examine short-term financial implications of many alternatives and test their sensitivity to changes in operating conditions.

The COMPA planning tests also emphasized an interesting implication of financing for financial policy. When receivables are factored there is a direct relationship between sales and the availability of "instant cash." Therefore, under adversity a firm must maintain a high level of sales to remain solvent. This causes a "treadmill" effect and the firm is forced to maximize sales with profit a secondary consideration. More conventional financing permits retrenching to conserve funds or increase profitability.

The extensive shadow price analysis in Chapter 5 gave empirical validity to the ability of linear programming to trace the opportunity locus of funds. The marginal revenue product functions that were defined not only demonstrated that they could be explicitly traced in an operating concern but also showed the impact of stipulated constraints on their slope and range. Shadow prices for funds derived from a fully integrated model are marginal values determined by the joint optimization of specified sources and uses. However, the formulation of the model limits the opportunities reflected in the shadow prices and they cannot normally be used to evaluate investment or financing decisions

completely external to the fully integrated model. In addition, the level of constraint and rigidities in the model affect the usefulness of the shadow prices. For this reason the best use of shadow prices for short-term financial planning is to structure the model so that it determines an optimal financing package. This approach eliminates direct consideration of shadow prices and precludes errors that may result from using them outside their area of applicability.

Before the shadow price of funds can be used without reservation in all investment decisions it will be necessary to develop fully integrated models that include all known opportunities for investing and financing during a long-term planning period. If a model of this sort can be developed it will alleviate most of the uncertainty associated with the use of existing capital budgeting techniques. However, this application will be left as a topic for additional research.

It is apparent from the forgoing discussion that the development of a fully integrated model may be an uncertain endeavor and it seems unlikely that the average manager can formulate one to represent his firm on a "do-it-yourself" basis. Building a fully integrated model is a complex undertaking that requires the services of well qualified (and expensive) people. For this reason, before a firm attempts to obtain such a model, the ultimate cost for both labor and computer services and the probability of success should be carefully weighed against expected benefits. Although this study has shown that linear programming can be used to materially expand the scope of practical financial analysis and planning by providing otherwise unobtainable information, the decision to develop a model must ultimately be based on management's assumption concerning the value of expected information.

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